

Fiscal Rules When Sovereign Debt Is Risky but Convenient

How Alternative Safe Collateral Shapes the Welfare Effects of Debt Brakes

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Motivation: a world of record debt — governed by uniform rules

- Global public debt exceeded \$100 trillion in 2024 and is projected to continue rising.

[View debt trends](#)

- Numerical fiscal rules are now widespread. The most prominent example is the EU's **Maastricht-style framework**, organized around reference values of 60% for debt and 3% for the deficit. Korea is considering a similar 60/3 rule.
- These common thresholds apply across economies in which government debt plays very different roles as a safe and liquid asset.
- **This paper:** the welfare effect of a debt brake depends on what other safe collateral investors can use.

Sovereign debt has two faces

1. It is risky

- More debt $\Rightarrow$ higher default risk, spreads, crisis exposure (Arellano 2008)
- Italy: CDS $\approx$ 79 bps, repeated crisis episodes
- This is the standard case *for* fiscal rules (Hatchondo et al. 2022).

2. It is convenient

- Government bonds provide collateral and liquidity services, so investors accept lower returns to hold them (Holmström and Tirole, 1998)
- Italian bonds: $\approx$ 32 bps of convenience value (Kaldorf–Röttger 2025)
- The sovereign therefore benefits from a convenience yield when safe collateral is scarce. (Krishnamurthy–Vissing-Jorgensen 2012)

What is the welfare effect of a debt brake when sovereign debt is both risky and valuable as collateral?

The missing ingredient: access to *outside* safe collateral is heterogeneous

A sovereign's convenience yield depends on the collateral substitutes available to its investors.

- That access varies across countries:
 - Euro-area investors can substitute toward Bunds and other eligible collateral.
 - In segmented or currency-specific markets, foreign safe assets are imperfect substitutes
 - For example, Korea's won collateral market is anchored by KTBs and MSBs.
- Where alternatives are scarce, the sovereign's debt *is* the safe asset and its scarcity rent is large — where alternatives abound, the rent is small

Research question

How does access to alternative safe collateral change the welfare effect of a debt brake?

This paper

What we do

- **Analytical model:** identifies the mechanism on how domestic ownership, collateral value and political myopia shape the welfare effect of a debt cap.
- **Quantitative analysis:** extends it to a calibrated infinite-horizon model and compares the debt brake and the spread brake for Italy and for a union whose members differ only in access to outside safe collateral.

Contribution

- A quantitative welfare evaluation of a debt brake when sovereign debt is both **defaultable** and **valued as collateral**.
- The optimal debt cap depends on access to outside safe collateral, so a common cap helps members whose debt is the scarce safe asset and hurts the rest. A spread brake dominates and travels across members.

Related literature

- **Sovereign debt, default, and debt dilution:** Eaton–Gersovitz (1981), Arellano (2008), Chatterjee–Eyigungor (2012), D’Erasmus–Mendoza (2016), Aguiar–Amador (2020, 2021).
- **Liquidity, collateral, and the value of public debt:** Krishnamurthy–Vissing-Jorgensen (2012), van Binsbergen–Diamond–Grotteria (2022), Angeletos–Collard–Dellas (2023), Kaldorf–Röttger (2025).
- **Fiscal rules and commitment:** Amador–Werning–Angeletos (2006), Halac–Yared (2014, 2018), Liu–Shen (2022), Hatchondo–Martinez–Roch (2022), Alfaro–Kanczuk (2026).

This paper studies fiscal rules when sovereign debt is both defaultable and socially valuable as collateral.

Analytical model: environment and objectives

Two periods, one-period bonds, exogenous revenue τy . The government issues B at $t = 0$ and repays or defaults at $t = 1$:

$$g_0 = \tau y_0 + q(B)B, \quad g_1^R = \tau y_1 - B, \quad g_1^D = \tau y_1,$$

$$V^R(B, y) = \psi_g v(\tau y - B) + s_h B, \quad V^D(y) = \psi_g [v(\tau y) - \phi(\tau y)].$$

- **Domestic holders:** a share s_h of the debt; they pay $s_h qB$ at issuance and receive $s_h B$ if repaid.
- **Collateral:** the bond provides collateral services while held; $\Theta(B)$ is total collateral.
- **Default** at $t = 1$ wipes out the debt: bondholders recover nothing.
- **Political myopia:** the government discounts with β_G , society with $\beta_S \geq \beta_G$.

$$\mathcal{W}(B; \beta) = \psi_g v(g_0) - s_h q(B)B + \beta s_h u(\Theta(B)) + \beta \mathbb{E}_y \max \{V^R(B, y), V^D(y)\}$$

Government: $B^G = \arg \max_B \mathcal{W}(B; \beta_G)$ Society evaluates: $\mathcal{W}_S(B^G) \equiv \mathcal{W}(B^G; \beta_S)$

[► Details](#)

Analytical model: default risk and the price of debt

Default is a low-income event

$$d = 1 \iff y < y^*(B), \quad p(B) = F(y^*(B))$$

$$\frac{dy^*}{dB} = \frac{\psi_g v'(\tau y^* - B) - s_h}{|\Delta_y(B, y^*)|}$$

Domestic ownership **weakens the debt–default link**: part of repayment is an internal transfer.

Credit value $\times$ convenience markup

$$q(B) = \frac{[1 - p(B)][1 + \Lambda(B)]}{1 + r_f}$$

$$\theta(B) = [1 - \kappa(p)](1 - p)B \quad \text{usable collateral}$$

$$\Theta(B) = \theta(B) + b_{\text{alt}} \quad \text{total collateral}$$

$$\Lambda(B) = [1 - \kappa(p)]u'(\Theta) \quad \text{convenience wedge}$$

Risk lowers the bond's value both as a **credit claim** and as **collateral**; outside safe collateral b_{alt} enters only through Θ .

Kaldorf–Röttger (2025)

► Default

► Pricing

Analytical model: the collateral channel

How much usable collateral does one more unit of debt create?

$$\theta'(B) = \underbrace{[1 - \kappa(p)](1 - p)}_{\text{quantity}} - \underbrace{Bp'(B) [\kappa'(p)(1 - p) + 1 - \kappa(p)]}_{\text{quality}}$$

If quality deteriorates enough, $\theta'(B) < 0$: **more face debt, less usable collateral.**

How does the convenience yield respond?

$$\Lambda'(B) = \underbrace{[1 - \kappa(p)] u''(\Theta) \theta'(B)}_{\text{collateral scarcity}} - \underbrace{\kappa'(p) p'(B) u'(\Theta)}_{\text{collateral haircut}}$$

A higher haircut makes each bond worse collateral, but by shrinking usable collateral it can make collateral *scarcer* and raise Λ .

► Details

Analytical model: S/D/L decomposition of welfare

Loosening a binding debt ceiling changes social welfare by

$$\frac{d\mathcal{W}_S}{dB} = \underbrace{S}_{\text{fiscal smoothing}} + \underbrace{D}_{\text{default / credit risk}} + \underbrace{L}_{\text{liquidity / collateral value}}$$

$$S = [\psi_g v'(g_0) - s_h] \frac{1-p}{1+r_f} - \beta_S \mathbb{E}[(1-d)\{\psi_g v'(g_1) - s_h\}]$$

$$D = -[\psi_g v'(g_0) - s_h] \frac{Bp'(B)(1+\Lambda)}{1+r_f}$$

$$L = [\psi_g v'(g_0) - s_h] \frac{(1-p)[\Lambda + B\Lambda'(B)]}{1+r_f} + \beta_S s_h u'(\Theta) \theta'(B)$$

- **S**: public goods today vs. the future repayment burden, net of transfers to domestic holders.
- **D**: debt-induced default risk lowers the value of the inframarginal debt stock ($D < 0$).
- **L**: collateral value of debt, through the bond price and direct domestic collateral utility.

The government's condition is the same with β_G in place of β_S .

► Derivation

Analytical model: when does a debt brake raise welfare?

At the government's choice B^G :

$$\left. \frac{d\mathcal{W}_S}{dB} \right|_{B^G} = (\beta_S - \beta_G) \left[\underbrace{s_h u'(\Theta) \theta'(B^G)}_{\text{collateral services}} - \underbrace{\mathbb{E}[(1-d)\{\psi_g v'(g_1) - s_h\}]}_{\text{repayment burden}} \right]$$

- $\beta_S = \beta_G$: B^G is socially optimal — no rule can raise welfare.
- $\beta_S > \beta_G$: if the repayment burden exceeds collateral services, a cap $\bar{B} < B^G$ raises welfare. The optimal cap solves $S + D + L = 0$.

Where outside safe collateral enters

Scarce outside collateral (low b_{alt}) raises $u'(\Theta)$. If $\theta'(B) > 0$, this enlarges the collateral term and **shrinks the gain from a cap**: society is reluctant to cut debt that serves as scarce collateral.

From the analytical model to the quantitative model

The analytical model signs the channels and shows one reason for a rule: **political myopia** ($\beta_G < \beta_S$). The quantitative model adds a second friction and puts numbers on both.

1. **Infinite horizon, Markov discretion.** The government re-optimizes every period and cannot commit to its future issuance.
2. **Long-term debt** ($\delta = 0.05$, five-year maturity). Today's bond price embeds the bond's future *resale* value $\Rightarrow$ future issuance **dilutes** legacy bondholders (Aguiar–Amador 2021).
3. **Stochastic, data-confrontable environment:** log income follows an AR(1); default entails exclusion and restructuring; CARA collateral demand makes the convenience yield decline with usable sovereign collateral and b_{alt} .

Two frictions a rule corrects

Myopia (analytical and quantitative) and **dilution** (quantitative only). Preserved: the pricing chain $q = m(1 + \Lambda)/(1 + r_f)$ and the forces (S), (D), (L).

Why Italy?

The experiment requires a sovereign whose debt is *simultaneously*

- **safe enough to be collateral:** Italian BTPs are ECB-eligible, widely repo'd, and carry a measurable convenience yield (≈ 32 bps) — the CY channel is *present*
- **risky enough to default:** CDS ≈ 79 bps, real crisis episodes — the default channel is *active*

This combination is rare

Core sovereigns (Germany, U.S.) have large convenience yields but negligible default risk — no fiscal-rule trade-off to study. Most emerging sovereigns have default risk but no collateral premium. Italy is one of the few economies where **both channels are quantitatively relevant at once** — exactly the environment the analytical model describes.

- Italy belongs to a currency union with common fiscal rules, motivating a common-rule counterfactual across heterogeneous member economies.
- Data and measurement follow [Kaldorf–Röttger \(2025\)](#), 2001–2012

Calibration

Parameter	Description	Value	Source / target
Externally set			
ρ_y, σ_y	Income persistence, volatility	0.937, 0.0092	Italian GDP
γ	Curvature of public-good utility	2	Conventional
r_f	Investor discount rate	0.0027	5-yr swap – inflation swap
δ, χ	Maturity, coupon	0.05, 0.0115	5-yr maturity; average BTP coupon
ω, ϑ	Recovery, restructuring probability	0.63, 0.08	Cruces–Trebesch (2013)
$\mu, \bar{\kappa}$	Haircut elasticity, maximum haircut	0.4, 0.7	Eurosystem collateral framework
s_h	Domestic share of sovereign debt	0.505	Domestic BTP holdings
b_{alt}	Outside safe collateral	4.34	Bunds outstanding / Italian GDP
β_S	Social discount factor	0.99	Hatchondo–Martinez–Roch (2022)
Jointly calibrated			
β_G	Government discount factor	0.976	Mean debt-to-GDP
d_0	Default cost, level	10.61	Mean CDS spread
ζ_1	Collateral service weight	122.4	Mean convenience yield
Held from an earlier calibration			
$d_1, \bar{g}, \zeta_2$	Default-cost slope, subsistence, CARA	50, 0.86, 1.50	sd of CDS, debt, CY
ψ_g	Weight on public-good utility	0.0174	Normalization

b_{alt} is the parameter varied across union members. β_S enters welfare only; the government decides with β_G .

Calibration: model fit

Moment	Data	Model
<i>Targeted</i>		
Mean debt QB/y	4.49	4.47
Mean CDS spread (bps)	79	85
Mean convenience yield (bps)	32	33
<i>Not re-targeted</i>		
sd debt	0.26	0.28
sd CDS spread (bps)	113	180
sd convenience yield (bps)	31	57
Mean sovereign spread (bps)	47	52
Default frequency (%/yr)	—	1.62

(β_G, d_0, ζ_1) are recalibrated to the three means; the volatility parameters $(d_1, \bar{g}, \zeta_2)$ are carried over and not yet re-targeted to the sd moments. Volatilities are overstated, as is common in this model class; sd(CY) is also raised by holding b_{alt} constant.

Two fiscal rules, one welfare criterion

Both rules follow [Hatchondo–Martinez–Roch \(2022\)](#); above the limit, debt must fall by ΔB (1% of GDP) per quarter.

Debt brake — a fixed level cap

$$B' \leq \max\{\bar{B}, B - \Delta B\}$$

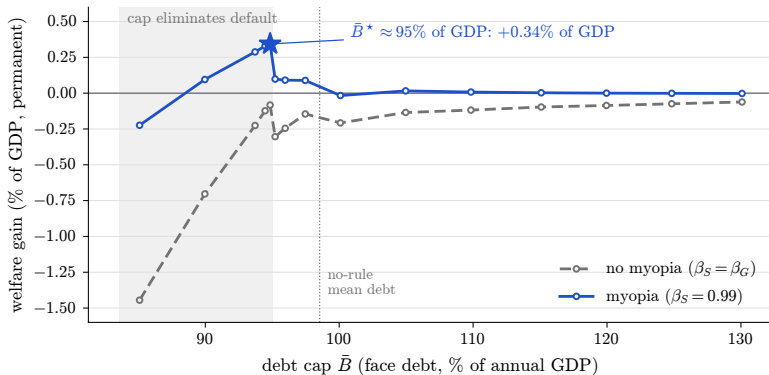
Spread brake — a cap that moves with risk

$$B' \leq \max\{\bar{B}_s(y), B - \Delta B\}$$

$$\bar{B}_s(y) = \max\{B' : \text{credit spread}(B', y) \leq \bar{s}\}$$

- **Welfare:** consumption-equivalent gain — the permanent transfer to households, in % of GDP every period, that leaves society indifferent. Evaluated with $\mathcal{W}_S$ ($\beta_S = 0.99$) at average debt and median income.
- **Optimal rules:** $\bar{B}$ and $\bar{s}$ chosen to maximize welfare — for each economy (*own rule*) or the union average (*common rule*).
- **Signal:** the brake reads the **credit spread** (default premium). The market spread = credit spread – convenience yield; reading it is the extension.

Result 1: Italy's optimal debt brake



- **Without myopia, no cap helps:** domestic ownership already disciplines default, so a cap mostly costs smoothing.
- **With myopia, the optimal cap is $\approx 95\%$ of GDP:** the loosest cap that still rules out default in recessions.

Result 2: debt brake vs. spread brake at the benchmark

Italy: optimal debt brake vs. the common spread brake.

		No rule	Debt brake ($\bar{B} = 95\%$)	Spread brake ($\bar{s} = 12.7$ bps)
(S)	mean face debt (% of GDP)	98.5	93.8	122.5
	rule binds (% of quarters)	—	84	4
	corr(public spending, output)	0.52	0.74	0.68
(D)	default (%/yr)	1.62	0.00	0.06
	default in adoption quarter (%) [†]	0.41	10.3	0.03
(L)	convenience yield (bps)	32.7	10.7	2.1
	usable sovereign collateral θ (% of GDP)	106	109	141
	welfare gain (% of GDP, permanent)		+0.34	+0.65
	from: public goods / domestic transfers		−0.48 / +0.13	+0.20 / −0.23
	default costs / collateral services		+0.64 / +0.05	+0.63 / +0.05

- Both rules remove default (**D**). The debt brake holds debt below its average: it binds 84% of the time and spending turns procyclical (**S**).
- The spread brake's ceiling **moves with income**: it rules out borrowing in distress, so debt *risks* at safe prices. The gain is similar for any threshold from 3 to 600 bps.

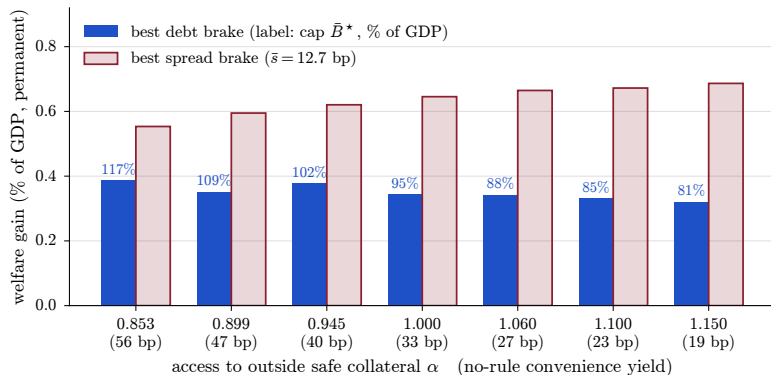
The union experiment

A stylized union of seven members, identical except in one respect: how much **foreign safe collateral** their investors can access, $b_{\text{alt},i} = \alpha_i \cdot 4.34$, $\alpha_i \in [0.853, 1.150]$, $\alpha = 1$ = calibrated Italy.

- Every other parameter is common — no member is separately recalibrated. b_{alt} is the *only* difference: a controlled experiment.
- Scarcer outside collateral $\Rightarrow$ a higher convenience yield on the member's own debt (no-rule CY from 56 to 19 bps), more debt, more default.
- Each member solved with its **own optimal debt cap**, a **common debt cap** that maximizes equal-weight union welfare, and one **common spread threshold**.

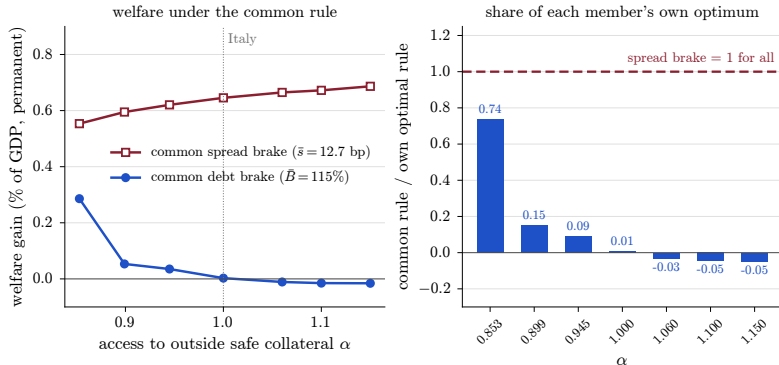
Which rule can a union share when its members' debt plays different collateral roles?

Result 3: each member's own optimal rule



- **The optimal debt cap depends on collateral access:** from 117% to 81% of GDP as access rises — each member's loosest cap that still rules out default.
- **The spread brake dominates for every member** (+0.55 to +0.69% of GDP vs. +0.32 to +0.39%), and its edge grows with access.

Result 4: one common rule for the whole union



- **A common debt brake fits few members.** The union-optimal cap (115%) sits next to the scarcest-collateral member's own (117%): a too-tight cap forces costly deleveraging on high-debt members; a too-loose one costs low-debt members little. It delivers 74% of that member's own gain; the others get at most 15%.
- **A common spread brake is portable:** one threshold delivers every member's own optimum.

Why: a level cap vs. a risk-based cap

Common rules, five representative members.

α	lower access		Italy	higher access	
	0.853	0.899	1.000	1.100	1.150
No-rule default (%/yr)	1.99	1.84	1.62	1.49	1.45
No-rule convenience yield (bps)	55.6	46.7	32.7	23.1	19.1
<i>Common debt brake ($\bar{B} = 115\%$ of GDP)</i>					
default (%/yr)	0.00	1.06	1.32	1.33	1.33
Δ mean face debt (pp of GDP)	−5.4	−4.2	−3.3	−2.4	−2.0
welfare (% of GDP)	+0.29	+0.05	+0.00	−0.02	−0.02
<i>Common spread brake ($\bar{s} = 12.7$ bps)</i>					
default (%/yr)	0.06	0.06	0.06	0.06	0.06
Δ mean face debt (pp of GDP)	+12.1	+16.6	+24.0	+29.4	+31.4
welfare (% of GDP)	+0.55	+0.59	+0.65	+0.67	+0.69

- A common level cap binds only where debt is high — the scarce-collateral members — and removes default only there.
- The spread brake adapts to each member's risk: default goes everywhere, and members with more outside collateral expand the safe-asset stock the most.

Extension: which spread should the brake read?

- So far the brake reads the **credit** spread — the compensation for default risk.
- Markets quote the **market** spread:

$$\text{market spread} = \text{credit spread} - \text{convenience yield}.$$

- Across the union, the no-rule convenience yield ranges from **19 to 56 bps** — larger than the 12.7 bps threshold itself.
- A common *market*-spread threshold is therefore looser for members whose debt is the scarcer safe asset: the signal is contaminated exactly where collateral is valuable.

Next step

Does the spread brake remain portable when its signal is a market spread?

Takeaways

1. **A rule needs a friction to fix.** With domestic holders, dilution alone leaves little for a debt brake to do; political myopia does. Italy's optimal cap is $\approx 95\%$ of GDP (+0.34% of GDP).
2. **A level cap pays only when it rules out recession defaults** — and adopting it below current debt triggers a default wave.
3. **The optimal cap depends on access to outside safe collateral** (117% to 81% of GDP), so one common cap helps scarce-collateral members and hurts the rest.
4. **A spread brake removes default without forced deleveraging:** one common threshold beats every member's own optimal debt cap, and the gain barely depends on the threshold.

Appendix: environment and objectives

- **Budget.** $g_0 = \tau y_0 + q(B)B$; at $t = 1$, $g_1^R = \tau y_1 - B$ if the government repays and $g_1^D = \tau y_1$ if it defaults. Default costs $\psi_g \phi(\tau y)$ in utility, with $\phi' \geq 0$.
- **Domestic holders.** A share s_h of the bond is held at home. $-s_h q(B)B$ is the domestic private-consumption cost of financing current issuance; $+s_h B$ in V^R means repayment is partly a transfer back to domestic residents.
- **Collateral services.** $\beta s_h u(\Theta)$ is the domestic value of the collateral services obtained while the bond is held. It is common to the repay and default branches because it accrues during the holding period.
- **Weights.** $\psi_g > 0$ is the social weight on public-good utility; it remains symbolic here.
- **Recovery.** Default at $t = 1$ wipes out the debt. Partial recovery, restructuring and re-entry are in the quantitative model.
- **Myopia.** Repay/default is decided ex post at $t = 1$, so $p(B)$, $q(B)$ and $\Theta(B)$ do not depend on β_G or β_S . Only the evaluation of the future differs. With $\beta_S = \beta_G$, $\mathcal{W}_S = \mathcal{W}_G$. A binding cap $\bar{B} < B^G$ gives welfare $\mathcal{W}_S(\bar{B})$.

Appendix: default incentives with domestic holders

Define the default incentive $\Delta(B, y) \equiv V^D(y) - V^R(B, y)$. Since v is concave and $\phi'(\tau y) \geq 0$,

$$\Delta_y = \psi_g \tau \left[v'(\tau y) - v'(\tau y - B) - \phi'(\tau y) \right] < 0,$$

so default is a **low-income event**: $d = 1$ iff $y < y^*(B)$, and $p(B) = F(y^*(B))$.

At the cutoff $\Delta(B, y^*) = 0$. With $\Delta_B = \psi_g v'(\tau y - B) - s_h$, the implicit-function theorem gives

$$\frac{dy^*}{dB} = -\frac{\Delta_B}{\Delta_y} = \frac{\psi_g v'(\tau y^* - B) - s_h}{|\Delta_y(B, y^*)|}, \quad p'(B) = f(y^*) \frac{dy^*}{dB}.$$

Interpretation

More debt raises default incentives when $\psi_g v'(\tau y^* - B) > s_h$. Domestic ownership weakens the debt–default link because part of repayment is an internal transfer rather than a payment abroad. Equivalently, the debt frontier $b^*(y)$ satisfies $db^*/dy = |\Delta_y|/(\psi_g v' - s_h)$.

Appendix: pricing risky but convenient debt

Investors are risk-neutral and value the bond as collateral. With zero recovery,

$$q(B) = \frac{[1 - p(B)][1 + \Lambda(B)]}{1 + r_f} \quad \text{credit value} \times \text{convenience markup.}$$

$$\theta(B) = [1 - \kappa(p(B))] [1 - p(B)]B,$$

$$\Theta(B) = \theta(B) + b_{\text{alt}},$$

$$\Lambda(B) = [1 - \kappa(p(B))] u'(\Theta(B)).$$

- θ : usable sovereign collateral
- b_{alt} : outside safe collateral
- $\kappa(p)$: haircut, increasing in default risk
- $u'(\Theta)$: marginal value of collateral
- Λ : convenience-yield wedge in the bond price

- Default risk enters twice: it lowers the expected payoff $(1 - p)$ and, through the haircut $\kappa(p)$, the bond's pledgeable value.
- Outside collateral b_{alt} affects the price only through $u'(\Theta)$: more of it makes collateral less scarce and lowers Λ .

Appendix: quantity, quality and the convenience yield

Differentiating $\theta(B) = [1 - \kappa(p(B))][1 - p(B)]B$ by the product rule,

$$\theta'(B) = \underbrace{[1 - \kappa(p)](1 - p)}_{\text{quantity effect}} - \underbrace{Bp'(B)[\kappa'(p)(1 - p) + 1 - \kappa(p)]}_{\text{quality effect}}.$$

- **Quantity:** one more unit of face debt supplies another collateral-producing claim.
- **Quality:** more debt raises default risk, lowering expected repayment and raising the haircut.

Differentiating $\Lambda(B) = [1 - \kappa(p(B))]u'(\Theta(B))$ with $\Theta'(B) = \theta'(B)$,

$$\Lambda'(B) = \underbrace{[1 - \kappa(p)]u''(\Theta)\theta'(B)}_{\text{collateral-scarcity effect}} - \underbrace{\kappa'(p)p'(B)u'(\Theta)}_{\text{collateral-haircut effect}}.$$

Scarcity ($u'' < 0$): if $\theta' > 0$, collateral becomes less scarce and Λ falls; if $\theta' < 0$, usable collateral shrinks, scarcity rises and Λ rises.

Haircut ($p' > 0$, $\kappa' \geq 0$): a higher haircut lowers the pledgeable fraction $1 - \kappa$; holding scarcity fixed, Λ falls.

Appendix: derivation of the S/D/L decomposition

Step 1 — current period. With proceeds $A(B) = q(B)B$,

$$\frac{d}{dB} [\psi_g v(g_0) - s_h qB] = [\psi_g v'(g_0) - s_h] \frac{d[q(B)B]}{dB}.$$

Step 2 — continuation. By the envelope argument (the moving-cutoff term is zero because $V^R(B, y^*) = V^D(y^*)$),

$$\frac{d}{dB} \mathbb{E} \max\{V^R, V^D\} = -\mathbb{E} \left[(1-d) \{ \psi_g v'(g_1) - s_h \} \right].$$

Step 3 — marginal revenue. From $q = (1-p)(1+\Lambda)/(1+r_f)$,

$$\frac{d[q(B)B]}{dB} = \frac{(1-p)(1+\Lambda) - Bp'(1+\Lambda) + (1-p)B\Lambda'}{1+r_f}.$$

Step 4 — collect terms. Add $\beta_S s_h u'(\Theta) \theta'(B)$ from the collateral-utility term. The $(1-p)(1+\Lambda)$ part of revenue splits into $(1-p)$, which joins the continuation term in S , and $(1-p)\Lambda$, which joins L ; the $-Bp'(1+\Lambda)$ part is D ; the $(1-p)B\Lambda'$ part joins L .

Under $p'(B) > 0$ and $\psi_g v'(g_0) > s_h$, $D < 0$. The government's first-order condition is the same expression with β_G in place of β_S .

Appendix: proof — value of a debt brake

The two objectives differ only in how the future is discounted:

$$\mathcal{W}_S(B) - \mathcal{W}_G(B) = (\beta_S - \beta_G) \left[s_h u(\Theta(B)) + \mathbb{E} \max\{V^R, V^D\} \right].$$

Differentiate, use Step 2 of the S/D/L derivation and $d\mathcal{W}_G/dB = 0$ at B^G :

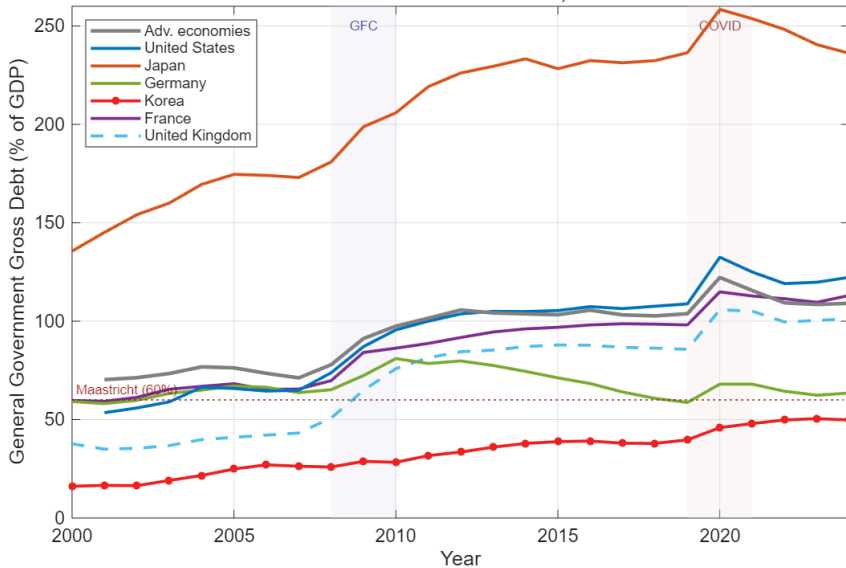
$$\left. \frac{d\mathcal{W}_S}{dB} \right|_{B^G} = (\beta_S - \beta_G) \left[s_h u'(\Theta) \theta'(B^G) - \mathbb{E}[(1-d)\{\psi_g v'(g_1) - s_h\}] \right].$$

- If the bracket is negative, $\mathcal{W}_S$ falls in B at B^G : a binding cap $\bar{B} < B^G$ raises welfare. The optimal cap solves $S + D + L = 0$ evaluated with β_S .
- **Outside collateral.** Holding B fixed, p and θ' do not depend on b_{alt} , so

$$\frac{\partial}{\partial b_{\text{alt}}} \left[s_h u'(\Theta) \theta'(B) \right] = s_h u''(\Theta) \theta'(B),$$

which is negative when $\theta'(B) > 0$. Less outside collateral raises the collateral term and shrinks the gain from a cap; if $\theta'(B) < 0$ (quality effect dominates), the sign reverses.

General Government Gross Debt, 2000--2024

[◀ Back to motivation](#)