Fiscal Rules When Sovereign Debt Is Risky but Convenient*

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Abstract

Investors value sovereign debt for its liquidity and as collateral even when it carries default risk. This paper studies how this convenience changes the optimal design of fiscal rules in a quantitative model of sovereign default calibrated to Italy. It compares a debt brake, which caps debt, with a spread brake, which stops new borrowing once the spread exceeds a threshold. Spread brakes raise welfare more than the best debt brake: their ceilings are loose when default risk is low and tighten when it rises. But the market spread, the spread observed in bond markets, reflects not only default risk but also the convenience yield. A spread brake that reads the market spread therefore loosens when demand for liquidity and collateral rises, and it raises welfare less than a spread brake based on default risk. The gap widens in a monetary union: members earn different convenience yields, so a common market-spread threshold allows each member a different level of default risk. Spread brakes should target default risk rather than market spreads.

JEL codes: E62, F34, G12, H63. **Keywords:** fiscal rules, sovereign default, convenience yield, spread brake.

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1 Introduction

Sovereign debt has risen in many countries over the past two decades. More than one hundred countries now limit government borrowing with fiscal rules. Since the Maastricht Treaty, the fiscal rules of the European Union have set the same reference values for every member: 60 percent of GDP for government debt and 3 percent for the deficit. The 2024 reform of these rules made the required adjustment path specific to each member but kept both values as a common anchor.[1] Korea has proposed rules built on the same values since 2020 but has not legislated one.[2]

Such caps treat sovereign debt as a source of risk. More debt raises the probability of default and raises the spread that lenders demand (Eaton and Gersovitz, 1981; Arellano, 2008). In models of sovereign default, fiscal rules are meant to limit this risk (Hatchondo et al., 2022). Sovereign debt, however, is also convenient. Investors value government bonds for their liquidity and as collateral, and they accept a lower yield to hold them (Holmström and Tirole, 1998; Krishnamurthy and Vissing-Jorgensen, 2012). The convenience yield is the amount by which a bond's yield falls short of what its default risk alone would imply. Italy, to which the model is calibrated and which is subject to the EU's common fiscal rules, illustrates both features. Over 2001–2012 the CDS premium on Italian sovereign debt averaged 79 basis points, and the convenience yield on Italian bonds averaged 32 basis points.[3] A rule that caps debt therefore also caps the supply of an asset that investors value.

The paper compares three fiscal rules: a debt brake and two versions of a spread brake. A debt brake caps the level of debt. A spread brake stops new borrowing once the interest spread on sovereign bonds exceeds a threshold (Hatchondo et al., 2022). Because the spread rises with default risk, the ceiling a spread brake places on debt is loose when risk is low and tight when risk is high. The two versions of the spread brake differ in which spread the threshold applies to. A market-spread brake sets the threshold on the spread observed in bond markets. A credit-spread brake sets it on the credit spread, which reflects default risk alone. When sovereign debt is convenient, the market spread equals the credit spread minus the convenience yield. Because the convenience yield rises with investors' demand for liquidity and collateral, a market-spread brake responds to that demand as well as to default risk. The paper answers the following questions. When sovereign debt is convenient, does a spread brake do better than a debt brake? To which spread should the threshold apply? Can one threshold serve several countries whose debt earns different convenience yields?

These questions are answered in two steps. First, a two-period model isolates the mechanism. In the model, government debt provides liquidity and collateral services, and domestic investors

[1] Before the reform, a member with debt above 60 percent of GDP had to reduce the excess by one twentieth a year on average. The reform replaced this requirement with a multi-year path for net expenditure set for each member.

[2] The 2020 proposal required (debt-to-GDP ratio / 60 percent) $\times$ (deficit-to-GDP ratio / 3 percent) ≤ 1. A 2022 revision caps the deficit-to-GDP ratio, excluding social security funds, at 3 percent, or at 2 percent when the debt-to-GDP ratio exceeds 60 percent. Neither has been passed by the National Assembly as of 2026.

[3] Following Kaldorf and Röttger (2025), I measure the convenience yield as the CDS premium minus the spread of the bond yield over the risk-free rate. Section 4 describes the data.

hold part of it. The government can default and is more impatient than society, a form of political myopia, so it borrows more than society would choose. The welfare effect of tightening a debt ceiling splits into three channels. The smoothing channel corresponds to the trade-off between spending today and repayment tomorrow. The default channel corresponds to the change in default risk. The liquidity channel corresponds to the change in the convenience yield and in the collateral services of the debt. Without a convenience yield, the liquidity channel is zero and the model reduces to a conventional sovereign-default model with only the smoothing and default channels. The liquidity channel makes a tighter debt ceiling more costly when investors hold less alternative safe collateral, because the sovereign's debt is then more valuable to them.

Second, a quantitative model measures the channels and compares the rules. It builds on the sovereign-default model with convenience yields and long-term debt of Kaldorf and Röttger (2025), extended to study fiscal rules and to include aggregate shocks to investors' demand for liquidity and collateral. With long-term debt, new borrowing dilutes existing bondholders, so a rule that limits borrowing has commitment value. The model is calibrated to Italy over 2001–2012, from the early years of the euro through the sovereign debt crisis.[4]

With each rule set to maximize welfare, three results emerge. First, spread brakes raise welfare more than a debt brake. Their ceilings track default risk, loose when risk is low and tight when it rises, whereas a debt brake binds in good times as well as bad. The cost of this flexibility is procyclicality: spread brakes tighten in recessions, when private consumption is already falling. The gain from tracking default risk outweighs the cost of procyclicality. Second, the threshold should be set on the credit spread rather than the market spread. The market spread falls when investors' demand for liquidity and collateral rises, even if default risk is unchanged. A market-spread brake therefore lets the government borrow more when demand is high and less when demand is low, whatever its default risk. Third, the advantage of the credit-spread brake is larger in a monetary union. Members' debt earns different convenience yields, so a common market-spread threshold allows each member a different level of default risk, while a common credit-spread threshold is the best rule for every member.

Related literature. The paper relates to three strands of literature. The first is the quantitative literature on sovereign default (Eaton and Gersovitz, 1981; Arellano, 2008; Aguiar et al., 2016). I use long-term debt as in Chatterjee and Eyigungor (2012), so that future issuance dilutes existing bonds. Hatchondo, Martinez, and Sosa-Padilla (2016) find that dilution accounts for most of the default risk in a model calibrated to Spain. They also study debt covenants, clauses in bond contracts that protect existing bondholders from dilution. Such covenants could help enforce fiscal rules. Aguiar and Amador (2020) show that dilution can also generate multiple equilibria. Domestic holdings of government debt are central in D'Erasmo and Mendoza (2016) and Sosa-Padilla (2018). Bocola et al. (2019) study a government that borrows from both domestic and foreign investors. They derive conditions under which total debt, rather than foreign-held debt alone, determines

[4]The sample ends in 2012 because the ECB's crisis interventions, beginning with the announcement of Outright Monetary Transactions, changed the pricing of euro-area sovereign bonds.

the incentive to default, and apply the model to the European debt crisis. In this paper domestic holdings matter because domestic holders enjoy the collateral services of the debt.

The second strand studies fiscal rules. Rules trade off commitment against flexibility (Amador et al., 2006; Halac and Yared, 2014). Azzimonti et al. (2016) assess balanced-budget rules in a political-economy model of fiscal policy. Halac and Yared (2018) compare deficit limits chosen jointly by a group of countries with limits chosen by each country. Dovis and Kirpalani (2020) ask whether fiscal rules reduce overborrowing by local governments when the central government cannot commit to enforce them. In models of sovereign default, Liu and Shen (2022) study fiscal commitment and default risk. Hatchondo et al. (2022) compare debt brakes and spread brakes. They find that a common spread brake generates larger gains than a common debt brake for a union of heterogeneous economies. Their model has no convenience yield, so the spread reflects only default risk and its risk premium. I ask what changes when the spread also reflects investors' demand for liquidity and collateral. Bianchi et al. (2023) show that under sovereign risk, fiscal austerity in a recession can be optimal, making sovereign risk a key driver of procyclical fiscal policy. The spread brakes in this paper are procyclical because their ceilings fall when default risk rises.

The third strand studies the liquidity and collateral value of government debt. Government bonds supply liquidity that private claims cannot (Holmström and Tirole, 1998), and the convenience premium on them falls with their supply (Krishnamurthy and Vissing-Jorgensen, 2012). Greenwood et al. (2015) study debt maturity when investors value the monetary services of short-term government debt. van Binsbergen et al. (2022) estimate the convenience yield on Treasuries from risk-free rates inferred from other asset prices. Passadore and Xu (2022) study the opposite force, an illiquidity premium that raises sovereign spreads. A macro-finance literature studies how the convenience value of government debt affects its valuation and the optimal or sustainable level of debt (Angeletos et al., 2023; Brunnermeier et al., 2024; Jiang et al., 2024; Mian et al., 2025). Choi et al. (2024) find that the prospect of losing its convenience yield in a default strengthens the incentive of the United States to repay and so raises its debt capacity. In the euro area, Jiang et al. (2026) find that convenience yields explain a large share of the variation in sovereign bond yields. They also find that convenience yields are larger for governments with stronger fiscal positions. Closest to this paper, Kaldorf and Röttger (2025) embed collateral services and risk-sensitive haircuts in a sovereign-default model calibrated to Italy. I extend their model to study fiscal rules.

Section 2 presents the two-period model and the decomposition into three channels. Section 3 presents the quantitative model and Section 4 the data and calibration. Section 5 presents the results. Section 6 concludes.

2 The analytical model

The analytical model is a two-period model of sovereign debt that investors value as collateral. The government values public goods, revenue is stochastic, bonds mature in one period, and default has

a utility cost.[5] Three features matter for fiscal rules. First, a share of the debt is held at home, so repayment is partly a transfer to domestic residents. Second, domestic holders enjoy the collateral services of the bond directly, so the convenience value of sovereign debt enters welfare and not only the bond price. Third, the government is more impatient than society, so it borrows more than society would choose. I refer to this as political myopia.

2.1 Environment

The government values public-good provision g with period utility $\psi_g v(g)$, where v is increasing and strictly concave with $v(g) \to -\infty$ as $g \to 0$, and $\psi_g > 0$ is the weight on public goods. Revenue is τy, where τ is a constant tax rate. Income y_0 is known; y_1 is drawn from a continuous distribution F with density $f > 0$. At $t = 0$ the government issues one-period bonds B at price $q(B)$; at $t = 1$ it repays or defaults. Default wipes out the debt and lowers the government's utility at $t = 1$ by $\psi_g \phi(\tau y_1)$, with $\phi' \geq 0$, without reducing its revenue.[6] The budgets are

$$g_0 = \tau y_0 + q(B)B, \qquad g_1^R = \tau y_1 - B, \qquad g_1^D = \tau y_1. \tag{1}$$

A share s_h of the bond is held by domestic financial intermediaries, which households own. These domestic holders pay $s_h q(B)B$ at issuance and receive $s_h B$ if the government repays. The repayment and default values at $t = 1$ are

$$V^R(B, y) = \psi_g v(\tau y - B) + s_h B, \qquad V^D(y) = \psi_g \big[v(\tau y) - \phi(\tau y)\big]. \tag{2}$$

While the bond is held, it provides collateral services. Let $\Theta(B)$ be total usable collateral, defined in Section 2.3. Domestic holders value these services at $s_h \zeta\, u(\Theta)$, where $\zeta > 0$ is investors' demand for liquidity and collateral (demand for collateral, for short) and u is increasing and strictly concave.[7]

The government discounts the future at β_G and society at $\beta_S \geq \beta_G$. For a discount factor β, write

$$\mathcal{W}(B; \beta) = \psi_g v(g_0) - s_h q(B)B + \beta\, s_h\, \zeta\, u\big(\Theta(B)\big) + \beta\, \mathbb{E}_y \max\big\{V^R(B, y), V^D(y)\big\}. \tag{3}$$

The government chooses $B^G = \arg\max_B \mathcal{W}(B; \beta_G)$. Society evaluates any debt level at $\mathcal{W}_S(B) \equiv \mathcal{W}(B; \beta_S)$. In this section, the fiscal rule is set as a debt ceiling: the government must choose $B \leq \bar{B}$. Because the decision to repay or default is taken at $t = 1$, the default probability, the price and the collateral stock do not depend on either discount factor; only the valuation of the future does.

[5] Treating the default cost as a utility cost follows Bianchi et al. (2018).

[6] Bondholders recover nothing after default. The quantitative model allows partial recovery, restructuring and re-entry.

[7] The term is common to the repayment and default branches because the services accrue during the holding period, before the payoff is realized.

2.2 Default

The government defaults when $\Delta(B, y) \equiv V^D(y) - V^R(B, y) > 0$.

Lemma 1 (Default cutoff)**.** *Default is a low-income event: $d = 1$ if and only if $y < y^*(B)$, and the default probability is $p(B) = F\big(y^*(B)\big)$. The cutoff responds to debt as*

$$\frac{dy^*}{dB} = \frac{\psi_g v'(\tau y^* - B) - s_h}{\big|\Delta_y(B, y^*)\big|}, \qquad p'(B) = f(y^*)\,\frac{dy^*}{dB}, \tag{4}$$

so more debt raises the default probability whenever $\psi_g v'(\tau y^* - B) > s_h$.

Appendix C gives the proof. The gain from defaulting falls with income, so default occurs at low income. Domestic ownership weakens the link between debt and default: part of repayment is an internal transfer rather than a payment abroad, so the marginal unit of debt raises the default incentive by less.

2.3 Pricing risky but convenient debt

As in the two-period model of Kaldorf and Röttger (2025, Section 3), the bond is priced by risk-neutral investors who also value it as collateral (Appendix A). The price is the credit price, which reflects default risk alone, times a convenience markup,

$$q(B) = \frac{\big[1 - p(B)\big]\big[1 + \Lambda(B)\big]}{1 + r_f}, \tag{5}$$

where Λ, the convenience yield, is determined by usable collateral. A haircut $\kappa(p)$, increasing in default risk, reduces the pledgeable fraction of each bond. Usable sovereign collateral, total collateral and the convenience yield are

$$\theta(B) = \big[1 - \kappa(p)\big](1 - p)B, \qquad \Theta(B) = \theta(B) + b_{\text{alt}}, \qquad \Lambda(B) = \big[1 - \kappa(p)\big]\,\zeta\,u'\big(\Theta(B)\big). \tag{6}$$

Risk lowers the bond's value twice: as a credit claim through $1-p$, and as collateral through $1-\kappa(p)$. Outside safe collateral b_{alt}, the supply of other safe collateral available to the same investors, enters only through Θ: more of it makes collateral less scarce and lowers the marginal value $\zeta u'(\Theta)$, hence Λ. To first order in p and Λ, the bond's spread over the risk-free rate is $p(B) - \Lambda(B)$: default risk net of the convenience yield.

2.4 Debt and usable collateral

How much usable collateral does one more unit of debt create? Differentiating θ,

$$\theta'(B) = \underbrace{\big[1 - \kappa(p)\big](1 - p)}_{\text{quantity}} - \underbrace{Bp'(B)\Big[\kappa'(p)(1 - p) + 1 - \kappa(p)\Big]}_{\text{quality}}. \tag{7}$$

The first term captures a quantity effect: each additional bond adds pledgeable collateral. The second captures a quality effect: more debt raises default risk, which lowers expected repayment and raises the haircut on the entire stock. If the quality effect dominates, $\theta'(B) < 0$, and more debt yields less usable collateral. Differentiating Λ in (6) links the convenience yield to (7):

$$\Lambda'(B) = \underbrace{\big[1-\kappa(p)\big]\,\zeta\,u''(\Theta)\,\theta'(B)}_{\text{collateral scarcity}} - \underbrace{\kappa'(p)\,p'(B)\,\zeta\,u'(\Theta)}_{\text{collateral haircut}}. \tag{8}$$

In (8), the first term captures a collateral scarcity effect: when debt adds usable collateral ($\theta' > 0$), collateral becomes less scarce and Λ falls, because $u'' < 0$. The second captures a collateral haircut effect: more debt raises default risk and the haircut, which lowers the pledgeable fraction of each bond and hence Λ. Default risk therefore enters Λ' twice. Through the haircut term it lowers Λ. Through θ', where the quality effect in (7) shrinks usable collateral, it makes collateral scarcer and raises Λ. The sign of $\Lambda'(B)$ depends on the relative strength of these effects.

2.5 The $S/D/L$ decomposition

Proposition 1 (Welfare decomposition)**.** *Let a debt ceiling $\bar{B} < B^G$ bind. The marginal effect of loosening it on social welfare is the sum of a smoothing channel S, a default channel D and a liquidity channel L,*

$$\frac{d\mathcal{W}_S}{d\bar{B}} = S + D + L, \tag{9}$$

$$S = \big[\psi_g v'(g_0) - s_h\big]\frac{1-p}{1+r_f} - \beta_S\,\mathbb{E}\Big[(1-d)\big\{\psi_g v'(g_1) - s_h\big\}\Big], \tag{10}$$

$$D = -\big[\psi_g v'(g_0) - s_h\big]\frac{\bar{B}\,p'(\bar{B})\,(1+\Lambda)}{1+r_f}, \tag{11}$$

$$L = \big[\psi_g v'(g_0) - s_h\big]\frac{(1-p)\big[\Lambda + \bar{B}\Lambda'(\bar{B})\big]}{1+r_f} + \beta_S\,s_h\,\zeta\,u'(\Theta)\,\theta'(\bar{B}), \tag{12}$$

all evaluated at $\bar{B}$. If $p' > 0$ and $\psi_g v'(g_0) > s_h$, then $D < 0$. If investors and domestic holders place no value on collateral ($\Lambda \equiv 0$ and $u \equiv 0$), then $L = 0$ and the decomposition reduces to $S + D$. The government's first-order condition is the same expression with β_G in place of β_S.

Appendix C gives the derivation. S is the smoothing channel: public goods today against the future repayment burden, net of the transfers to domestic holders. D is the default channel: the marginal unit raises default risk and lowers the value of the entire inframarginal stock. In a conventional sovereign-default model, this channel favors a tighter rule. L is the liquidity channel. Its first term is the convenience revenue on the marginal unit, Λ, plus $\bar{B}\Lambda'$, the change in the convenience yield on the existing stock when the sovereign issues more. Its second term is the direct value to domestic holders of the collateral services the marginal unit creates. Outside safe collateral b_{alt} enters the decomposition through the convenience yield Λ and the marginal value of collateral $\zeta u'(\Theta)$.

2.6 When does a debt brake raise welfare?

Proposition 2 (Political myopia and the value of a debt brake)**.** *At the government's choice B^G,*

$$\left.\frac{d\mathcal{W}_S}{dB}\right|_{B^G} = (\beta_S - \beta_G)\Big[\underbrace{s_h\,\zeta\,u'(\Theta)\,\theta'(B^G)}_{\text{collateral services}} - \underbrace{\mathbb{E}\big[(1-d)\{\psi_g v'(g_1) - s_h\}\big]}_{\text{repayment burden}}\Big]. \tag{13}$$

(i) If $\beta_S = \beta_G$, no binding ceiling raises welfare.

(ii) If $\beta_S > \beta_G$, tightening a ceiling marginally below B^G raises welfare if and only if the repayment burden exceeds the collateral services of the marginal unit. An interior optimal ceiling solves $S + D + L = 0$.

(iii) Holding debt fixed at B^G, $\partial\big[s_h\,\zeta\,u'(\Theta)\,\theta'(B^G)\big]/\partial b_{\text{alt}} = s_h\,\zeta\,u''(\Theta)\,\theta'(B^G)$, which is negative when $\theta'(B^G) > 0$.

Part (i) holds because, with $\beta_S = \beta_G$, $\mathcal{W}_S$ coincides with the government's objective. Part (ii) states the trade-off at the government's choice: the only reason to cut debt is that the government undervalues the future repayment burden, and the only reason not to is the collateral the marginal bond provides. Part (iii) shows how outside collateral matters. When outside collateral is scarce, the sovereign's debt is valuable collateral, so the collateral term is large and a cap gains less. If the quality effect dominates ($\theta' < 0$), the sign reverses.[8]

2.7 To which spread should the threshold apply?

A spread brake lets the government borrow up to the debt level at which a spread reaches a threshold. This debt level is the brake's *ceiling*. Two spreads are available. The *credit spread* compensates investors for default risk alone; in the model it is $p(B)$. The *market spread* is the yield over the risk-free rate that markets quote: by (5), approximately $p(B) - \Lambda(B)$. A credit-spread brake applies its threshold to the first; a market-spread brake applies it to the second.

To see when the choice matters, suppose that demand for collateral ζ is not known when the rule is set. By (6), a higher ζ raises the convenience yield at every level of debt. Demand is realized at $t = 0$, before the debt choice, and is unrelated to the sovereign's fiscal position, so it does not enter the credit spread. The data support this. Over 2001–2007, about three quarters of the variation in the convenience yields of Italy, Spain, Portugal and Greece is common to all four countries, and convenience yields rose together in the 2008–10 flight to safety (Section 4).

Because the market spread is the credit spread minus the convenience yield, a market-spread brake with threshold $\bar{s}^M$ stops borrowing at the ceiling $\bar{B}^M$ at which the credit spread reaches an effective threshold,

$$\bar{s}^M + \Lambda\big(\bar{B}^M, \zeta\big). \tag{14}$$

[8] Part (iii) is a partial derivative at a given debt level. The total effect of b_{alt} on the value of a rule also runs through the government's choice B^G, which moves with b_{alt}; the quantitative model computes the total effect (Section 5.5).

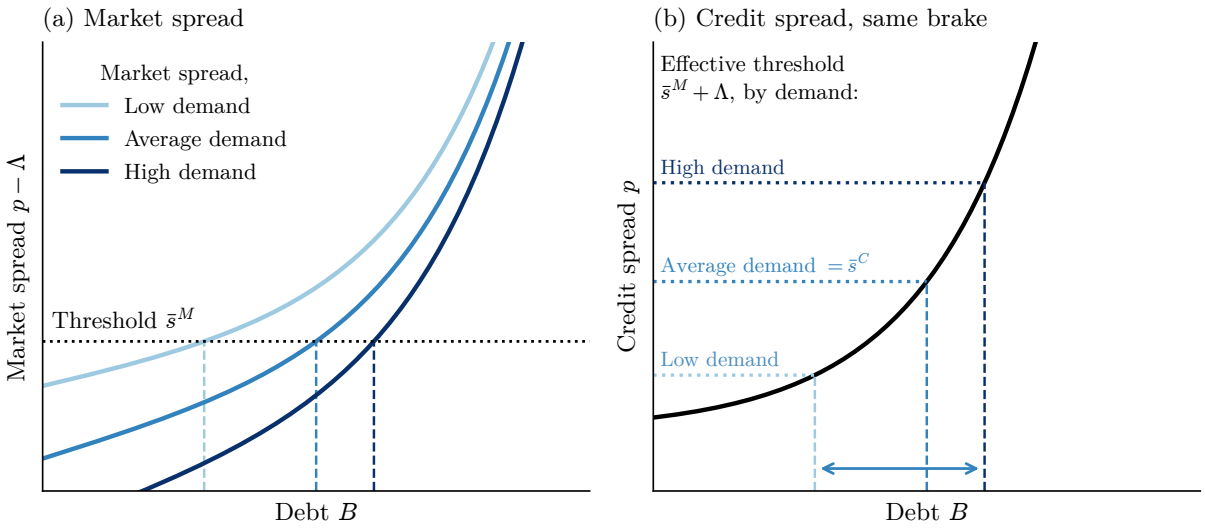


Figure 1: One market-spread brake, drawn two ways. Panel (a): a fixed threshold on market-spread curves that shift with demand for collateral. Panel (b): the same brake as a credit-spread brake. The credit-spread curve does not move with demand; the effective threshold, $\bar{s}^M + \Lambda$, does. Both give the same three ceilings. By contrast, a credit-spread brake with constant threshold $\bar{s}^C$, drawn at the average-demand line, has the same ceiling at every level of demand.

Proposition 3 (Market-spread and credit-spread brakes)**.** *A market-spread brake with threshold $\bar{s}^M$ is a credit-spread brake with the state-dependent threshold (14). It is equivalent to a credit-spread brake with a constant threshold $\bar{s}^C$ if and only if the convenience yield at its ceiling, $\bar{\Lambda}$, is the same in every state; then $\bar{s}^C = \bar{s}^M + \bar{\Lambda}$. In particular, the two brakes are equivalent when the convenience yield does not depend on demand.*

Figure 1 draws the same market-spread brake in two ways. Panel (a) plots the market spread against debt for three levels of demand. The threshold is fixed and the market-spread curve shifts down when demand rises, so the ceiling moves out. Panel (b) plots the credit spread against debt. The curve is fixed and the effective threshold (14) rises with demand. For each level of demand, the effective threshold crosses the curve at the same ceiling as in panel (a). By contrast, a credit-spread brake with constant threshold $\bar{s}^C$ has the same ceiling at every level of demand, because demand does not enter the credit spread. The market-spread ceiling moves with demand by

$$\frac{d\bar{B}^M}{d\zeta} = \frac{\partial\Lambda/\partial\zeta}{p'(\bar{B}^M) - \partial\Lambda/\partial B}, \tag{15}$$

which is positive when the market spread rises with debt. It is the shift in the market-spread curve divided by its slope (Appendix D).

Figure 1 shows what a market-spread brake does differently. A brake exists to limit default risk, but the default risk this brake allows moves with investors' demand for collateral, which does not change the sovereign's default risk. When demand is high, the brake lets the government borrow up

to a higher credit spread. When demand falls, it forces the government to stop borrowing, although default risk has not risen. This movement helps only if the best ceiling moves with demand by about as much. Higher demand does raise the value of debt through the liquidity channel in Proposition 1, so the best ceiling may rise with demand. But by (15), the market-spread ceiling moves by the shift in the convenience yield divided by the slope of the market-spread curve, and nothing ties this to the gain in the liquidity channel. The two-period model therefore does not rank the two brakes. Section 5.3 shows that in the quantitative model the market-spread ceiling moves far more with demand than the credit-spread ceiling, and the credit-spread brake does better at every level of mean debt.

3 The quantitative model

The analytical model signs the channels and shows one reason for a rule: the government is more impatient than society ($\beta_G < \beta_S$). It cannot value a rule or compare the two spread brakes quantitatively. The quantitative model keeps the analytical model's pricing of risky but convenient debt, its domestic bondholders and its three channels, and adds the features needed for a calibration to Italy. The government lives forever and re-optimizes every period without commitment. Debt is long term, so a bond's price today depends on how much the government will issue tomorrow. Future issuance dilutes existing bondholders, a friction a rule can correct in addition to the government's impatience (Aguiar and Amador, 2021). Income is persistent, default leads to exclusion and restructuring, the government chooses taxes as well as public spending, and investors' demand for collateral moves over time.

3.1 Environment

Time is quarterly. Income follows a log AR(1),

$$\ln y' = \rho_y \ln y + \sigma_y \varepsilon', \qquad \varepsilon' \sim N(0,1). \tag{16}$$

Investors' demand for collateral ζ is common to all issuers and independent of income. Its logarithm follows an AR(1) with persistence ρ_ζ and unconditional standard deviation σ_ζ,

$$\ln \zeta' - \mu_\zeta = \rho_\zeta \big(\ln \zeta - \mu_\zeta \big) + \sigma_\zeta \sqrt{1-\rho_\zeta^2}\, \eta', \qquad \eta' \sim N(0,1), \tag{17}$$

where μ_ζ is set so that the mean demand level is $\bar{\zeta}$. The exogenous state is $s = (y, \zeta)$. The endogenous states are the government's face debt B and whether it has access to markets.

Debt. A unit of debt matures each period with probability δ and pays a coupon χ while outstanding (Chatterjee and Eyigungor, 2012), so debt service per unit is $\nu = \delta + (1-\delta)\chi$. Net borrowing

when the government chooses next-period debt B' is

$$R(B, B', s) = q(B', s)\big[B' - (1-\delta)B\big] - \nu\, B. \tag{18}$$

Fiscal block. The government raises a lump-sum tax T and provides public goods g; households consume c. Domestic intermediaries, which households own, hold a share s_h of the debt, so households finance that share of net borrowing:

$$g = T + R, \qquad c = y - T - s_h R. \tag{19}$$

Given net borrowing R and income y, the government chooses the tax to maximize households' flow utility:

$$U(R, y) = \max_{T \in [0, y]} \ \psi_g\, v(g) + \theta_c\, u_c(c), \qquad v(g) = \frac{(g - \bar{g})^{1-\gamma} - 1}{1-\gamma}, \qquad u_c(c) = \frac{c^{1-\sigma}}{1-\sigma}. \tag{20}$$

Public goods have a subsistence level $\bar{g}$. As in Hatchondo et al. (2022), households cannot borrow or save, and the government sets fiscal policy to maximize their utility over private and public consumption. With a lump-sum tax and no labor choice, households make no decisions of their own, so U is their utility given the government's choices. In the analytical model the tax rate is fixed, so the burden of repayment falls on public goods alone. Here the government can also shift it to private consumption. The weight ψ_g sets how much households value public goods relative to private consumption, and Section 4 calibrates it to the ratio of public spending to GDP. The weight θ_c sets the units of utility: $\theta_c u_c'(\bar{c}) = 1$ at average private consumption $\bar{c}$. Domestic holders value collateral services in units of consumption, as the investors in Appendix A do, so θ_c converts these services into utility at the marginal utility of average private consumption.[9]

Default. A government that defaults stops servicing and issuing debt and loses output $L(y) = \max\{0, \lambda_0 y + \lambda_1 y^2\}$, which rises more than proportionally with income (Chatterjee and Eyigungor, 2012). In default, net borrowing is zero and income falls to $y - L(y)$, so the flow payoff is (20) evaluated at these values: $U_d(y) = U\big(0,\, y - L(y)\big)$. Each quarter it regains market access with probability ϑ, with face debt restructured to ωB.

Collateral services. As in Section 2, domestic holders value the collateral services of the debt they hold. Their valuation is

$$h(\Theta, \zeta) = s_h\, \zeta\, u(\Theta), \qquad u(\Theta) = \frac{1 - e^{-\zeta_2 \Theta}}{\zeta_2}, \tag{21}$$

[9] This follows Krishnamurthy and Vissing-Jorgensen (2012), in whose model convenience services add to consumption and are valued at the marginal utility of consumption.

where Θ is total usable collateral, defined below. Demand scales the marginal value of collateral, $\zeta u'(\Theta) = \zeta e^{-\zeta_2 \Theta}$, which falls with the stock of collateral at the rate ζ_2.

3.2 Pricing

Investors are risk neutral and value the bond as collateral, as in Section 2.3 (Appendix A). Let $d(B', s')$ be next period's default probability. The one-quarter default probability on a bond issued at B' and the haircut are

$$p(B', s) = \mathbb{E}_{s'|s}\, d(B', s'), \qquad \kappa(B', s) = \min\left\{p(B', s)^{\mu},\ \bar{\kappa}\right\}, \tag{22}$$

where μ is the elasticity of the haircut to default risk and $\bar{\kappa}$ the maximum haircut. The expected payoff of a new bond includes its resale value,

$$m(B', s) = \mathbb{E}_{s'|s}\Big[(1 - d')\big\{\nu + (1 - \delta)\, q(B'', s')\big\} + d'\, q_d(B', s')\Big], \tag{23}$$

where B'' is next period's debt choice and q_d is the price of defaulted debt, defined below. The term $q(B'', s')$ captures dilution: a government that issues more tomorrow lowers the resale value of today's bonds, and lenders price that in. Usable collateral, the convenience yield and the price are

$$\Theta(B', s) = \big[1 - \kappa(B', s)\big]\, m(B', s)\, B' + b_{\text{alt}}, \tag{24}$$

$$\Lambda(B', s) = \big[1 - \kappa(B', s)\big]\, \zeta\, e^{-\zeta_2 \Theta(B', s)}, \tag{25}$$

$$q(B', s) = \frac{m(B', s)\big[1 + \Lambda(B', s)\big]}{1 + r_f}, \tag{26}$$

where b_{alt} is outside safe collateral, as in Section 2.3. Equation (25) has the structure of (6): Λ is the pledgeable fraction times the marginal value of collateral $\zeta u'(\Theta)$ from (21). The same function therefore governs the bond price and the domestic holders' valuation.

The demand state ζ and the scarcity slope ζ_2 play different roles. Demand moves the convenience yield of every issuer together, as the common factor in the data does. The scarcity slope sets how much the convenience yield falls when collateral, including the sovereign's own debt, becomes more abundant. Together with the volatility of demand, it sets how far the ceiling of a market-spread brake moves with demand (15). Section 4 estimates it.

Defaulted debt. A defaulted bond pays nothing while the government is excluded. Each quarter, with probability ϑ, it is restructured into ω units of performing debt; otherwise it stays in default. Its expected payoff and usable collateral are

$$m_d(B, s) = \vartheta\, \omega\, m(\omega B, s) + (1 - \vartheta)\, \mathbb{E}_{s'|s}\, q_d(B, s'), \qquad \Theta_d(B, s) = (1 - \bar{\kappa})\, m_d(B, s)\, B + b_{\text{alt}}, \tag{27}$$

and its price q_d follows from (25)–(26) with m_d, Θ_d and the maximum haircut $\bar{\kappa}$ in place of m, Θ and κ.

Spreads. The credit price q_P is the price of a bond with the same default risk and no convenience value: it solves (23) and (26) with $\Lambda = 0$ and q_P in place of q. For any price, the annualized spread over the risk-free rate is

$$\text{spr}(q) = 10^4\Big[\big(1 + \nu/q - \delta\big)^4 - (1 + r_f)^4\Big] \tag{28}$$

in basis points. The credit spread is $\text{spr}(q_P)$, the model counterpart of the CDS premium. The market spread is $\text{spr}(q)$. The convenience yield is their difference, $\text{spr}(q_P) - \text{spr}(q)$.

3.3 Government problem

With market access, the values of repaying and choosing B' and of defaulting are

$$V^R(B, B', s) = U\big(R(B, B', s), y\big) + \beta_G\, h\big(\Theta(B', s), \zeta\big) + \beta_G\, \mathbb{E}_{s'|s} V(B', s'), \tag{29}$$

$$V^D(B, s) = U_d(y) + \beta_G\, h\big(\Theta_d(B, s), \zeta\big) + \beta_G\, \mathbb{E}_{s'|s}\Big[\vartheta\, V(\omega B, s') + (1 - \vartheta)\, V^D(B, s')\Big], \tag{30}$$

and the government chooses the best option,

$$V(B, s) = \max\Big\{V^D(B, s),\ \max_{B' \in \Gamma(B,s)} V^R(B, B', s)\Big\}. \tag{31}$$

Usable collateral Θ and Θ_d are defined in (24) and (27). $\Gamma(B, s)$ is the set of debt choices the fiscal regime allows (Section 3.4); without a rule it contains every feasible B'. The government decides with β_G; society evaluates with β_S (Section 3.6). Each option carries a small i.i.d. taste shock, which makes the government's choices smooth in the state (Appendix H).

3.4 Fiscal rules

The rules follow Hatchondo et al. (2022) with one simplification. A rule sets a ceiling on next-period face debt, and a government already above the ceiling may keep its debt but not increase it.[10]

Debt brake. A fixed ceiling, $\Gamma(B, s) = \{B' : B' \le \max\{\bar{B},\ B\}\}$.

Spread brake. A ceiling that moves with a spread. The government may choose any B' at which the spread on the new bond does not exceed the threshold $\bar{s}$, or any B' that does not increase its debt:

$$\Gamma(B, s) = \big\{B' :\ \text{spr}\big(\tilde{q}(B', s)\big) \le \bar{s}\big\}\ \cup\ \big\{B' : B' \le B\big\}. \tag{32}$$

[10] Hatchondo et al. (2022) also require a government above the ceiling to reduce its debt by a fixed amount each quarter. Requiring a reduction of 1 percent of annual GDP per quarter leaves the debt brake's welfare gain unchanged to two decimals and lowers the credit-spread brake's gain from 0.85 to 0.82 percent of consumption.

The credit-spread brake applies the threshold to the credit spread, $\tilde{q} = q_P$; the market-spread brake applies it to the market spread, $\tilde{q} = q$. Because the spread rises with debt, the debt levels that satisfy the threshold form an interval, so the brake sets a ceiling $\bar{B}^s(s)$ that moves with income and with demand.

Lenders price debt under the future borrowing and default policies that the rule induces, so a rule raises the price of debt today by limiting future dilution.

3.5 Equilibrium

The government cannot commit to future policies, so the equilibrium is Markov rather than Ramsey: policies and prices depend only on face debt B, the exogenous state s and market access.

Definition. Given a fiscal regime Γ, a recursive equilibrium consists of value functions V, V^R and V^D; policies for default, borrowing and taxes; and price schedules q, q_P and q_d, such that:

(i) given the price schedules, the value functions satisfy (29)–(31), the tax attains the maximum in (20), and the default and borrowing policies are the choice probabilities implied by the taste shocks;

(ii) given the policies, the price schedules satisfy (22)–(26), the credit price q_P is computed as above, and defaulted debt is priced by (27). These conditions are the investors' first-order conditions when their holdings equal the debt issued (Appendix A);

(iii) the feasible set $\Gamma(B, s)$ is evaluated at the equilibrium price schedules.

Condition (iii) matters only for the spread brakes, whose feasible set (32) depends on q_P or q: a spread brake changes the prices that define its own ceiling. Long-term debt models can have several equilibria (Aguiar and Amador, 2020). The equilibrium computed here is the limit of the finite-horizon economy, and solving from other starting points gives the same equilibrium (Appendix H).

3.6 Welfare

Society uses the government's flow payoffs, including the holders' collateral services, and discounts at $\beta_S \geq \beta_G$. Let $\{g_t, c_t, \Theta_t\}$ be the allocation the government chooses under a fiscal regime, given demand ζ_t, and let

$$W(\lambda) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_S^t \Big[\psi_g v(g_t) + \theta_c \, u_c\big((1+\lambda)c_t\big) + \beta_S \, s_h \, \zeta_t \, u(\Theta_t) \Big] \tag{33}$$

be social welfare when private consumption is scaled by $1+\lambda$ at every date and in every state. The welfare gain of a rule is the consumption equivalent λ that solves

$$W^{\text{no rule}}(\lambda) = W^{\text{rule}}(0), \tag{34}$$

the permanent percentage increase in private consumption that would make society as well off without the rule as with it (Lucas, 1987). Both sides start from the mean debt of the economy without a rule, with median income and median demand.

4 Data and calibration

4.1 Italian spreads, 2001–2012

The model is calibrated to Italy at a quarterly frequency. Financial moments are from the dataset of Kaldorf and Röttger (2025) for 2001Q1–2012Q4: five-year CDS premia, five-year government bond yields, five-year swap rates as the risk-free benchmark, and government debt relative to quarterly GDP.[11] The market spread is the bond yield minus the swap rate. The convenience yield is the CDS premium minus the market spread, which in the model is $\mathrm{spr}(q_P) - \mathrm{spr}(q)$.[12] Fiscal moments are from Eurostat quarterly government accounts, over the longest available sample, 1999Q1–2019Q4. Appendix B describes the series. The same financial series for Spain, Portugal and Greece are used to estimate the scarcity slope and the common component of convenience yields.

Figure 2 plots the three series. In 2001–2007 the CDS premium averaged 9 basis points and Italian bonds yielded less than the swap rate, so the convenience yield, about 20 basis points, exceeded the CDS premium. From 2008 the convenience yield rose to about 100 basis points: in 2009Q1 the market spread was 66 basis points against a CDS premium of 171. In 2011–12 the convenience yield fell and turned negative, so the market spread rose above the CDS premium. A market-spread brake would have responded to these movements in the convenience yield as well as to those in the CDS premium.

4.2 Parameters set outside the model

Table 1 lists the parameters. Income follows an AR(1) process estimated on Italian GDP. The risk-free rate is the real five-year swap rate, 0.27 percent per quarter. Debt has an average maturity of five years ($\delta = 0.05$), and the coupon matches the average coupon on five-year Italian Treasury bonds. After default, debt is restructured with recovery $\omega = 0.63$ (Cruces and Trebesch, 2013), and the government regains market access with probability $\vartheta = 0.08$ per quarter, an average exclusion of three years. The haircut schedule follows the Eurosystem's collateral framework. Private domestic residents hold half of Italian government debt securities ($s_h = 0.505$). Outside safe collateral b_{alt} is German government debt relative to Italian quarterly GDP. Society discounts at $\beta_S = 0.99$. The weight on private consumption, θ_c, sets the marginal utility of private consumption to one at

[11] All financial series are quarterly averages of daily Bloomberg data, and the swap rate is the euro interest-rate swap rate. Debt is Eurostat consolidated gross government debt with debt securities at market value, and its model counterpart is the market value of debt, qB'/y.

[12] This treats the CDS premium as pricing default risk alone, as in Kaldorf and Röttger (2025). In practice it also reflects the risk that the seller of protection fails to pay and the liquidity of the CDS market; Chaumont et al. (2024) model how CDS trading interacts with sovereign debt markets.

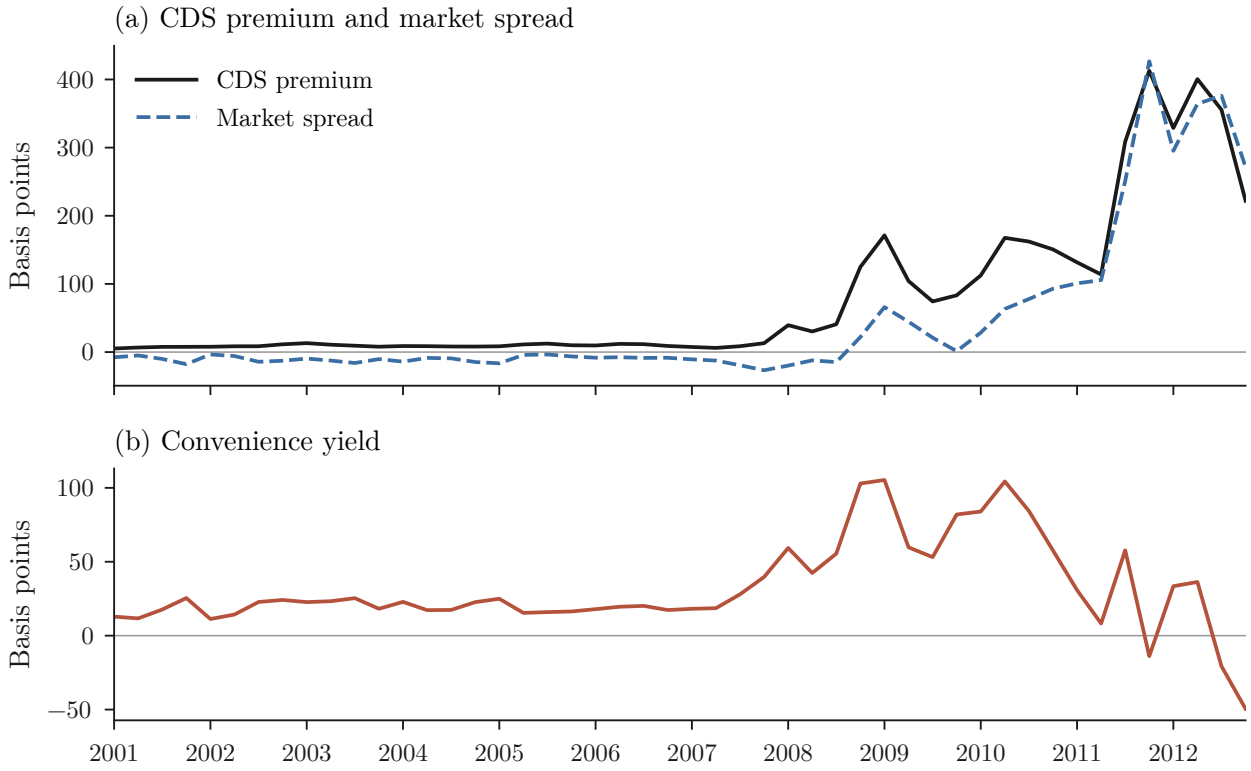


Figure 2: Italian spreads, 2001Q1–2012Q4, in basis points. Panel (a): CDS premium and market spread; panel (b): convenience yield.

its average level. This measures utility in units of private consumption and values the domestic holders' collateral services at the marginal utility of average consumption (Section 3).

The scarcity slope ζ_2, which sets how much a country's convenience yield falls when its own debt rises, is hard to identify in the data. Euro-area convenience yields differ little across countries with very different debt: in 2001–2007 Spain's debt was half of Italy's relative to GDP, and its convenience yield was 24 basis points against Italy's 20. A regression of the log convenience yield on debt for Italy, Spain, Portugal and Greece over 2001Q1–2011Q3, with country and time effects and a control for default risk, gives a point estimate of 0.18 for the scarcity slope (Appendix B). The scarcity slope is set to $\zeta_2 = 0.20$, close to this estimate. Holding the haircut fixed, an increase in usable collateral of 25 percent of annual GDP then lowers the per-period convenience value Λ in (25) by about 18 percent. The convenience yield on a five-year bond falls by less, by 3 to 7 percent when debt rises from 100 to 125 percent of annual GDP, because it also prices convenience in later quarters, when debt has returned toward its usual level.

4.3 Parameters calibrated jointly

Nine parameters are chosen jointly to minimize the distance between eleven model moments and their data counterparts (Table 2). Each is mainly identified by one or two moments. The government's discount factor β_G sets mean debt. The public-good weight ψ_g sets the ratio of public

Table 1: Parameters

	Description	Value	Source or target
Set outside the model			
$\rho_y,\ \sigma_y$	Income persistence, volatility	0.937, 0.0092	Italian GDP
r_f	Risk-free rate (quarterly)	0.0027	Real swap rate
$\delta,\ \chi$	Maturity, coupon	0.05, 0.0115	Five-year maturity; average coupon
ω	Recovery	0.63	Cruces and Trebesch (2013)
ϑ	Re-entry probability	0.08	Three-year exclusion
$\mu,\ \bar{\kappa}$	Haircut elasticity, maximum haircut	0.4, 0.7	Eurosystem collateral framework
s_h	Domestic share of debt	0.505	Private domestic holdings
b_{alt}	Outside safe collateral	4.34	German debt / Italian quarterly GDP
ζ_2	Scarcity slope	0.20	Panel of euro-area sovereigns
γ	Curvature, public goods	2	Kaldorf and Röttger (2025)
σ	Curvature, private consumption	2	Hatchondo et al. (2022)
θ_c	Weight on private consumption	0.639	$\theta_c u_c'(\bar{c}) = 1$
β_S	Social discount factor	0.99	Hatchondo et al. (2022)
Calibrated jointly			*Most informative moment*
β_G	Government discount factor	0.9807	Mean debt
ψ_g	Weight on public goods	1.89×10^{-4}	Public spending / GDP
$\bar{g}$	Subsistence public goods	0.1729	Cyclicality of public spending
λ_0	Output loss, linear term	-0.642	Output loss; CDS mean and volatility
λ_1	Output loss, quadratic term	0.737	Jointly with λ_0
$\bar{\zeta}$	Mean demand for collateral	0.00446	Mean convenience yield
σ_ζ	Volatility of demand	0.743	Calm volatility of convenience yield
ρ_ζ	Persistence of demand	0.844	Persistence of German convenience yield
σ_ε	Scale of taste shocks	0.0131	Volatility of debt

The lower panel is chosen jointly to match the eleven targeted moments in Table 2; each row lists the moment that parameter affects most.

spending to GDP, and the subsistence level $\bar{g}$ sets how much public spending moves with output. The output-loss parameters λ_0 and λ_1 set the output loss in default and the mean and volatility of the CDS premium. The parameters of the demand process set the convenience yield: $\bar{\zeta}$ sets its mean, σ_ζ its volatility in calm quarters, and ρ_ζ the persistence of the German convenience yield, which moves with demand alone because German default risk is close to zero. The scale of the taste shocks affects the volatility of debt.

The model matches the level and the calm-period behavior of the convenience yield and the level of default risk. It misses two features of the full sample. First, the volatility of Italy's convenience yield is 6 basis points in the model and 31 in the data. In the data, almost all of this volatility comes from 2008–2012: a common rise in convenience yields during the 2008–10 flight to safety and a fall in Italy's own convenience yield in 2011–12. The volatility of demand is calibrated to calm periods; Appendix F studies episodes of this size separately. Second, the CDS premium is below 20 basis points in 58 percent of quarters in the data but almost never in the model, where default risk stays positive in every state.

Table 2: Model fit

Moment	Data	Model
Targeted		
Mean debt / quarterly GDP	4.59	4.62
Mean CDS premium (bp)	79	83
Volatility of CDS premium (bp)	113	102
Volatility of debt / quarterly GDP	0.24	0.27
Mean convenience yield (bp)	32	31
Public spending / GDP	0.193	0.188
Elasticity of public spending to output	0.154	0.177
Output loss in default	0.10	0.08
Volatility of convenience yield, calm (bp)	5.8	5.7
Persistence of German convenience yield	0.85	0.84
Common component of convenience yields, calm	0.78	0.96
Not targeted		
Volatility of convenience yield, full sample (bp)	31	6
Mean market spread (bp)	47	52
Correlation of convenience yield and CDS	0.07	0.41
Share of quarters with CDS below 20 bp	0.58	0.00
Correlation of public spending / GDP and output	−0.62	−0.55
Default frequency (% per year)	–	1.82

Financial moments and debt 2001Q1–2012Q4; other fiscal moments 1999Q1–2019Q4. Calm: 2001Q1–2007Q4. Common component: share of the calm-period variance of convenience yields explained by common time effects for Italy, Spain, Portugal and Greece. Debt at market value. The output loss in default is a target, not a data moment (Appendix B).

5 Results

All welfare gains are consumption equivalents (34) relative to the economy without a rule, in percent of permanent private consumption.

5.1 Debt brakes and spread brakes

Both spread brakes are worth more than the best debt brake, and the credit-spread brake is worth the most (Table 3). Without a rule, debt is 101 percent of GDP, the credit spread averages 83 basis points and the annual probability of default is 1.8 percent.

Every rule lowers default risk, which raises welfare through the default channel D. The rules differ in how they do it. The best debt brake caps debt at 101 percent of GDP and binds in 62 percent of states: the government gives up borrowing in good times as well as in bad, which lowers welfare through the smoothing channel S. It lowers default from 1.8 to 0.03 percent per year and is worth 0.58 percent of consumption. The credit-spread brake is worth 0.85 percent and lowers default from 1.8 to 0.2 percent per year. Its ceiling moves with income and binds when default risk rises, so it lowers default risk while letting debt rise to 117 percent of GDP. The credit spread falls from 83 to 7 basis points, so the government borrows more, and more cheaply. The higher debt also lowers the convenience yield, from 31 to 26 basis points. The liquidity channel L is small

Table 3: Fiscal rules at the benchmark

	No rule	Debt brake (101.2%)	Credit-spread brake (7.5 bp)	Market-spread brake (−12.5 bp)
Welfare gain (% of consumption)	–	0.58	0.85	0.82
public goods	–	0.00	0.02	0.02
private consumption	–	0.56	0.78	0.76
collateral services	–	0.02	0.05	0.05
Mean debt (% of GDP)	101.1	96.9	116.5	116.8
Mean ceiling (% of GDP)	–	101.2	121.8	122.5
Default (% per year)	1.82	0.03	0.17	0.26
Mean credit spread (bp)	83.1	1.3	6.6	10.5
Mean convenience yield (bp)	31.1	30.4	25.5	25.4
Share of states where the rule binds	–	0.62	0.53	0.51

Welfare at mean debt, median income and median demand of the economy without a rule. Debt at face value; its market value without a rule is 115 percent of GDP. The debt brake's ceiling is shown at median income; the spread brakes' mean ceilings are averages over each brake's stationary distribution. The rule binds when the government would choose debt above the ceiling.

under every rule, at most 0.06 percent of consumption: the debt brake barely changes debt, and the extra debt under the spread brakes earns a lower convenience yield. The convenience yield matters instead for the choice of spread to which the threshold applies (Section 5.3).

Under every rule, almost all of the welfare gain reaches households through private consumption; public goods and collateral services contribute little (Table 3). Public goods add at most 0.02 percent of consumption. Because public goods and private consumption have the same curvature, the government keeps the ratio of c to $g - \bar{g}$ fixed and splits additional resources in that ratio. The subsistence level $\bar{g}$, set to match the low cyclicality of public spending, leaves $g - \bar{g}$ at under 2 percent of c, so almost all additional resources go to private consumption. Collateral services to domestic holders add 0.05 percent of consumption under the spread brakes, which hold more debt, and 0.02 under the debt brake.

5.2 The gains by channel

Lower default risk accounts for much of every rule's gain, and the debt brake and the spread brakes differ mainly through the smoothing channel S (Table 4). The smoothing channel S corresponds to the change in net borrowing from foreign investors, valued at the bond prices of the economy without a rule net of their convenience value. The default channel D corresponds to the higher price that lenders pay for the debt because default risk is lower, plus the output losses avoided by defaulting less. The liquidity channel L corresponds to the convenience revenue on the debt issued plus the collateral services of the debt held at home. As in Proposition 1, S values borrowing without its convenience value, and L collects that value. Each change in resources is valued at the marginal utility of consumption in the quarter and state in which it occurs and converted into consumption equivalents. Appendix I gives the details.

The price gain in D and the smoothing channel S largely offset each other. A rule that lowers

Table 4: The welfare gains by channel

	Debt brake	Credit-spread brake	Market-spread brake
Welfare gain (% of consumption)	0.58	0.85	0.82
S: smoothing	−0.58	−0.33	−0.30
D: default	1.01	1.04	0.99
higher price of debt	0.55	0.59	0.57
avoided output loss	0.47	0.44	0.42
L: liquidity	0.02	0.06	0.06
convenience revenue	0.01	0.01	0.01
holders' collateral services	0.02	0.05	0.05
Residual	0.13	0.08	0.08

Consumption equivalents at mean debt, median income and median demand of the economy without a rule. The residual is the welfare gain minus $S + D + L$ (Appendix I). Rows may not add because of rounding.

default risk raises the price of its debt. Lenders are risk neutral, so the credit price of a bond equals the expected value of its repayments. The government therefore pays for that higher price by repaying in states in which it would otherwise have defaulted, and this extra repayment lowers S. For example, if a rule lowers the default probability on a bond from 2 percent to zero, the bond sells for about 2 percent more, which raises D, but the government now also repays it in the 2 percent of states in which it would have defaulted, which lowers S by about as much. For the debt brake the two nearly cancel, 0.55 against −0.58, so its gain comes mainly from the avoided output loss, 0.47 percent of consumption. The credit-spread brake avoids almost as much output loss, 0.44, and gains a further 0.26 from the price gain and S together, because its ceiling lets the government borrow more when default risk is low. It gives up borrowing mainly in recessions, when its ceiling falls (Section 5.4). Consumption is low then, so each unit of lost borrowing costs more welfare: if every quarter were weighted equally, its loss through S would be 0.08 smaller. The liquidity channel adds 0.06 percent of consumption under the spread brakes, most of it the collateral services of the extra debt held at home, and 0.02 under the debt brake. The market-spread brake falls short of the credit-spread brake through D because it allows more default: 0.26 against 0.17 percent per year (Table 3). It loses 0.05 percent of consumption through D and recovers 0.03 through S.

5.3 Credit spread or market spread

The credit-spread brake does better than the market-spread brake at every level of mean debt (Figure 3). Thresholds that apply to different spreads cannot be compared directly. The figure therefore plots each threshold against its mean debt: the mean of the stationary distribution of the debt-to-GDP ratio under that threshold. With each brake at its best threshold, the market-spread brake is worth about 0.025 percent of consumption less than the credit-spread brake.

The market-spread brake does worse because its ceiling moves with demand for collateral even when default risk does not, while the credit-spread ceiling barely moves (Figure 4). Panels (a) and (b) are the quantitative model's version of Figure 1. A rise in demand raises the convenience yield, which shifts the market spread down at every level of debt (panel a). With a threshold of

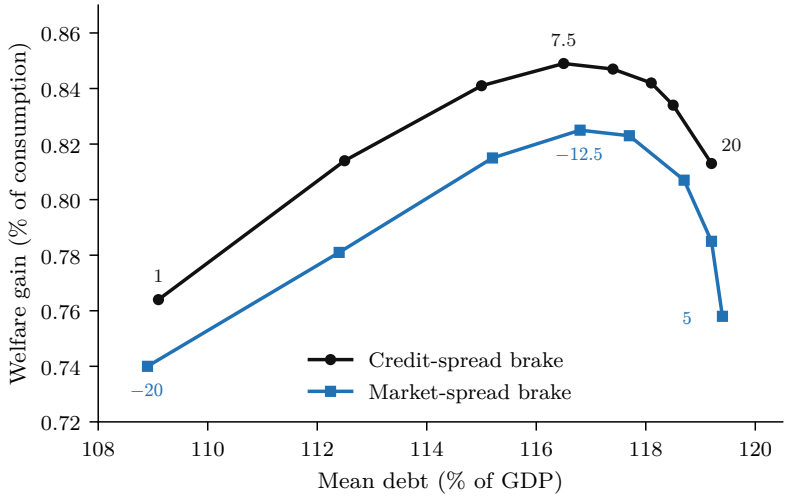


Figure 3: Welfare gain of each brake, in percent of consumption, against the mean debt it produces, in percent of GDP. Each point is one threshold; labels give selected thresholds in basis points.

−12.5 basis points, the market-spread ceiling at median income therefore rises from 118.5 percent of GDP when demand is low to 125.9 percent when it is high. The threshold is negative because the convenience yield exceeds the credit spread at the ceiling. In terms of the credit spread, the same ceilings lie at 7.5 to 19.5 basis points at median income (panel b). The ceiling of the credit-spread brake, with a threshold of 7.5 basis points, moves only from 121.2 to 121.8 percent of GDP at median income (panel c), because under this brake the credit spread barely depends on demand.

Unlike in the two-period model, the credit spread under the market-spread brake depends on demand for collateral even at the same debt: at 112 percent of GDP it is about 8 basis points when demand is high and about 6 when it is low (Figure 4b). The credit spread prices the debt the government will carry in later quarters, not only its current debt. High demand loosens the market-spread ceiling, and demand is persistent, so a government facing high demand is expected to borrow more in the quarters that follow, and lenders price the added risk of default and dilution. The gap closes at high debt, where the risk of default on the current debt dominates. Under the credit-spread brake the ceiling barely moves with demand, so expected future borrowing, and with it the credit spread, is nearly the same at every level of demand (panel c).

The market-spread brake loosens when demand for collateral is high and tightens when it is low, while the credit-spread ceiling, which tracks default risk, barely moves across the same demand states, from 121.6 to 122.1 percent of GDP (Table 5). When demand is high, the market-spread brake lets the government borrow more at a higher default risk: the credit spread on its debt is 13.5 basis points, against 6.5 under the credit-spread brake. When demand is low, it is tighter: 26 percent of states lie above the ceiling and must hold or reduce debt, against 19 percent under the credit-spread brake. Private consumption in these quarters is 80.0 percent of output, against 80.6. Relative to the credit-spread brake, the market-spread brake gains 0.11 percent of consumption in quarters of high demand and 0.03 in quarters of median demand, but loses 0.17 in quarters

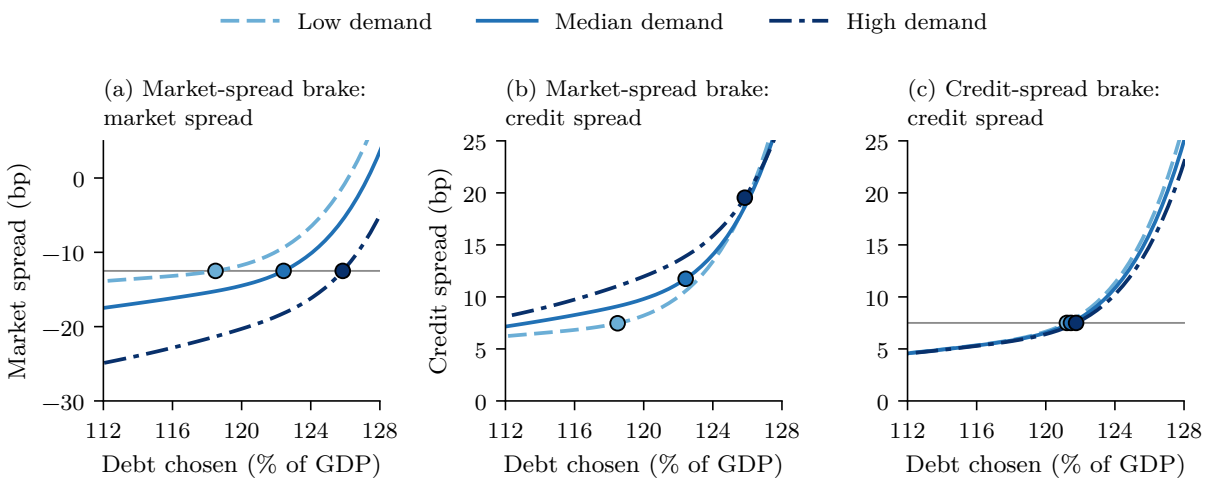


Figure 4: Spread schedules and ceilings at median income, by demand for collateral. Dots mark the ceilings; gray lines mark the thresholds. Panels (a) and (b): market-spread brake with a threshold of −12.5 basis points; panel (a) plots its market spread and panel (b) its credit spread. Panel (c): credit-spread brake with a threshold of 7.5 basis points.

of low demand. Part of that loss pays for the extra borrowing in high demand: when demand falls, the ceiling tightens, and the government must stop borrowing and service the debt it raised earlier. The three demand states cover ordinary variation in demand; a larger, unanticipated fall moves the market-spread ceiling much further. Appendix F simulates a fall in demand to zero for eight quarters. The market-spread ceiling falls on impact from 122.6 to 84 percent of GDP, by 38 percentage points. Default risk does not change, so the credit-spread ceiling does not move (Figure 8). The market-spread brake then stops all new borrowing, and the welfare gap between the two brakes widens by 0.014 percent of consumption.

5.4 Recessions

The spread brakes are procyclical. They let debt rise in normal times, so the economy enters a recession with more debt, and their ceilings fall as default risk rises. Private consumption therefore falls more than without a rule or under the debt brake. Figure 5 averages the paths around large recessions, with every economy simulated under the same shocks.[13] At the trough income is 5.8 percent below its mean (panel a).

Before the recession the spread brakes hold debt of 117 percent of mean GDP, against 100 without a rule and 97 under the debt brake. As income falls, default risk rises and the spread brakes' ceilings fall with it: at the trough the credit-spread ceiling is 107 percent of mean GDP, against 123 eight quarters earlier, and 69 percent of histories are at or above it. Debt must stop

[13] A large recession is a quarter in which income is at least two standard deviations below its mean and lower than in the eight quarters before and after it, and in which income was at or above its median in at least one of the preceding eight quarters. The simulation has 500 independent histories of 5,000 quarters each, after a burn-in of 500 quarters, with the same income and demand shocks in every economy. Episodes with a default in the eight quarters before the trough are excluded, so the number of episodes differs across economies, from 1,125 to 1,594.

Table 5: The two brakes by demand for collateral

	Credit-spread brake			Market-spread brake		
Demand for collateral	Low	Median	High	Low	Median	High
Effective credit threshold (bp)	7.5	7.5	7.5	7.6	12.2	20.5
Ceiling (% of GDP)	121.6	121.8	122.1	118.6	122.6	126.1
Debt (% of GDP)	116.3	116.5	116.7	115.0	116.9	118.4
Credit spread on debt (bp)	6.7	6.6	6.5	7.8	10.3	13.5
Default (% per year)	0.20	0.17	0.12	0.22	0.27	0.29
Share of states above the ceiling	0.19	0.18	0.17	0.26	0.17	0.12
Private consumption (% of output)	80.6	80.7	80.8	80.0	80.7	81.2
Market minus credit-spread brake (% of consumption)				−0.17	0.03	0.11

Low, median and high demand are the three nodes of the demand process, with stationary weights 0.25, 0.5 and 0.25 (Appendix H). Averages over income and debt in each demand state, at each brake's stationary distribution. The effective credit threshold is the credit spread at the ceiling. Credit-spread brake at 7.5 basis points, market-spread brake at −12.5. The last row gives the market-spread brake's welfare gain minus the credit-spread brake's in quarters of each level of demand, starting from the stationary distribution of the economy without a rule.

growing, and the adjustment falls on private consumption, which drops at the trough from 81 to 76 percent of output under both spread brakes, against 79 without a rule and 80 under the debt brake. Because they enter a recession with more debt, the spread brakes also leave some default risk. The probability of default next quarter peaks at 3.5 percent under the credit-spread brake and 4.2 percent under the market-spread brake, against 12.7 percent without a rule and 0.2 percent under the debt brake. Of these recessions, 12 percent end in default within four years under the credit-spread brake and 16 percent under the market-spread brake, against 39 percent without a rule and less than 1 percent under the debt brake. After the trough the spread brakes' ceilings rise again and consumption recovers within four quarters.

The welfare gains in Table 3 include this loss of consumption in recessions. Splitting the gains by the income of the quarter in which they accrue shows that the credit-spread brake loses to the debt brake in low-income quarters and gains more in the rest (Table 6).[14] The debt brake gains most in quarters of below-average income, when default would otherwise occur. The credit-spread brake gains most in quarters of average and above-average income, when its ceiling lets the government borrow at low spreads. In the lowest-income quarters it gains almost nothing, because its ceiling falls and private consumption drops. Against the debt brake, the credit-spread brake loses 0.33 percent of consumption in the quarters more than half a standard deviation below mean income, a third of all quarters, and gains 0.58 in the rest. The two spread brakes differ little in any income bin. The ranking also holds whatever income is when the rule is adopted (Appendix G).

The ranking is the same under Italy's income and demand shocks of 2007–2012 (Appendix E). Adopted in 2007Q1, the credit-spread brake is worth 0.86 percent of consumption, the market-spread brake 0.83 and the debt brake 0.65. The debt brake is worth more than at the benchmark

[14] All economies start from the stationary distribution of the economy without a rule and follow their own paths. Each quarter's difference in flow utility, converted to consumption equivalents as in (34), is assigned to one of five bins by that quarter's income, with cutoffs at ±0.5 and ±1.5 standard deviations of log income. As in Table 5, flows are assigned to quarters, not to decisions.

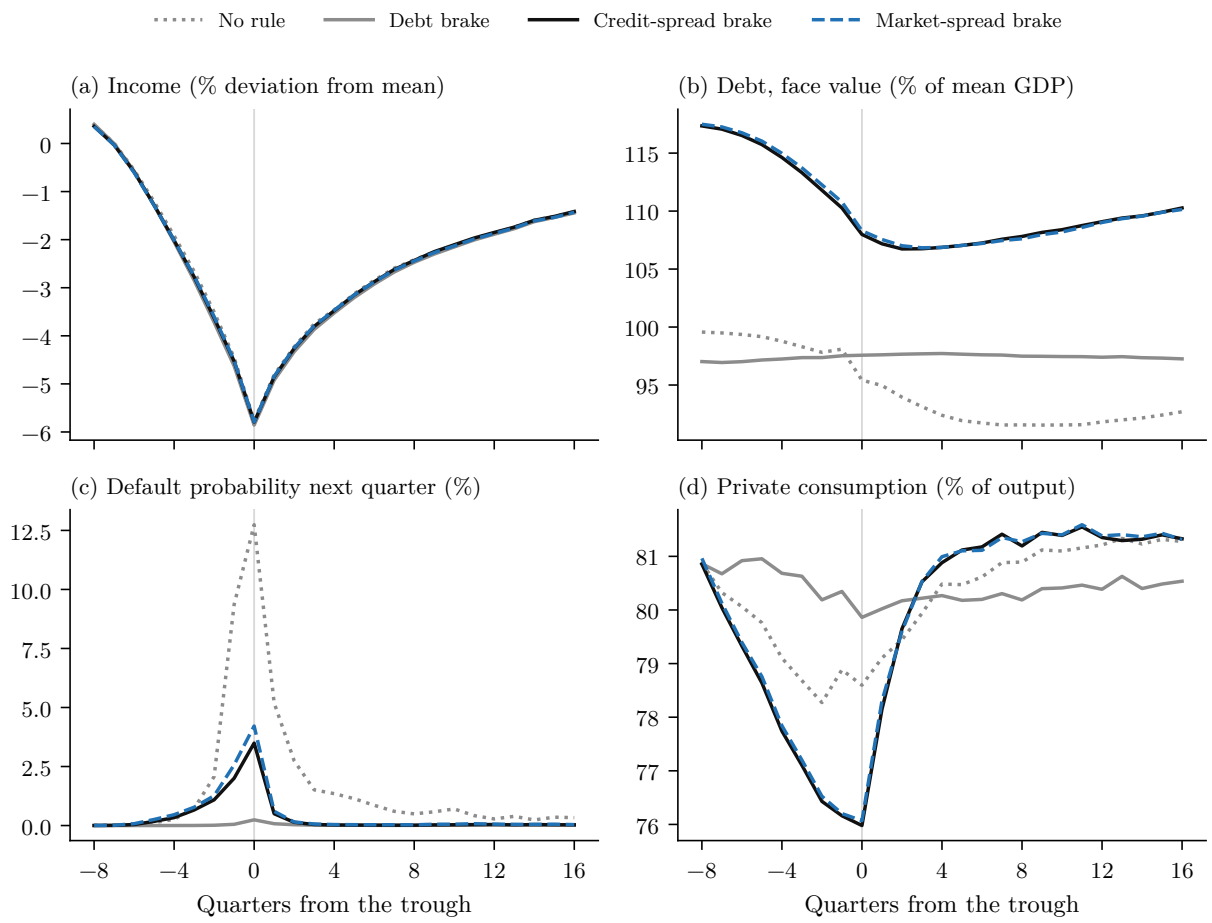


Figure 5: Average paths around large recessions in simulated data. Quarter 0 is the trough. Debt and the default probability are averages over histories with market access in that quarter.

because of the realized shocks, not the starting state: from Italy's 2007Q1 state, with shocks drawn from the model, it would be worth 0.55. The realized path adds 0.10, against 0.05 for each spread brake, because it contains nine quarters of income more than half a standard deviation below its mean, in which the debt brake gains most.

5.5 Fiscal rules in a monetary union

In a monetary union whose members differ in outside safe collateral, one credit-spread brake is the best rule for every member, whereas under a common debt brake or market-spread brake default risk differs across members (Table 7 and Figure 6). Members share one fiscal framework but differ in how much outside safe collateral their investors hold, in addition to their own government's bonds. The union consists of members identical to Italy except in outside safe collateral, $b_{\text{alt}} = 4.34\,\alpha$, with α from 0.6 to 1.4; $\alpha = 1$ is Italy. Each rule is evaluated against the member's own economy without a rule. A common rule applies one threshold to every member, set to maximize the members' average welfare gain.

Table 6: The welfare gains by income (% of consumption)

Income (sd from mean)	< -1.5	-1.5 to -0.5	-0.5 to 0.5	0.5 to 1.5	> 1.5	Total
Share of quarters	0.07	0.25	0.35	0.25	0.07	1
Debt brake	0.15	0.36	0.15	−0.05	−0.03	0.58
Credit-spread brake	0.02	0.16	0.30	0.27	0.09	0.84
Market-spread brake	0.01	0.17	0.30	0.25	0.08	0.81
Credit-spread − debt brake	−0.13	−0.19	0.15	0.32	0.12	0.26
Credit-spread − market-spread brake	0.00	−0.01	0.00	0.02	0.01	0.02

Gains against the economy without a rule unless stated. Totals are welfare gains starting from the stationary distribution of the economy without a rule, so they differ slightly from Table 3, which starts at its mean debt. The bins sum to the total up to a term of at most 0.007, because the consumption equivalent is not linear in welfare. Differences are computed before rounding.

Scarcer outside collateral raises the convenience yield on a member's own debt, from 22 basis points at $\alpha = 1.4$ to 43 at $\alpha = 0.6$. Without a rule, the member with the least outside collateral borrows more than the member with the most, 103 against 99 percent of GDP, at almost the same default rate. Because the convenience yield on its debt is higher, this member pays a lower interest rate on its debt, and the collateral services it would lose in default make repayment more valuable. It can therefore carry more debt at the same risk.

Debt brakes. The best common debt ceiling is 101 percent of GDP, Italy's own best. It is too tight for members with scarce outside collateral, whose debt is worth more to investors as collateral: at $\alpha = 0.6$ it is worth 0.53 percent of consumption, against 0.58 for the member's own best ceiling of 103.5 percent. It is too loose for members with abundant outside collateral, which borrow less without a rule. At $\alpha = 1.4$ it allows default of 0.30 percent per year, against 0.04 under the member's own best ceiling of 99.7 percent, and it is worth about 0.04 percent of consumption less. Every debt brake, common or not, is worth at least 0.25 percent of consumption less than the credit-spread brake.

The credit-spread brake. The best threshold of the credit-spread brake is 7.5 basis points for every member, so a common threshold costs no member anything. Because the threshold applies to the credit spread, which reflects default risk alone, every member under it defaults with the same frequency, 0.17 percent per year (Figure 6b). The brake's gain varies little across members, from 0.87 percent of consumption at $\alpha = 0.6$ to 0.83 at $\alpha = 1.4$, mainly because the collateral services of its debt are worth less to members with more outside collateral.

The market-spread brake. The best common threshold of the market-spread brake is -10 basis points. Because the market spread is the credit spread minus the convenience yield, this threshold allows a higher credit spread in members whose debt earns a higher convenience yield. Default under the brake is 0.49 percent per year at $\alpha = 0.6$ and 0.17 at $\alpha = 1.4$. Relative to the credit-spread brake, the common market-spread brake costs 0.07 percent of consumption at $\alpha = 0.6$,

Table 7: Fiscal rules in a union whose members differ in outside safe collateral

Outside collateral, α	0.6	0.8	1.0	1.2	1.4
No rule					
Convenience yield (bp)	42.8	36.5	31.1	26.4	22.4
Debt (% of GDP)	103.4	102.1	101.1	100.2	99.4
Default (% per year)	1.80	1.81	1.82	1.83	1.84
Welfare gain (% of consumption)					
Credit-spread brake, 7.5 bp	0.87	0.86	0.85	0.84	0.83
Market-spread brake, common (−10 bp)	0.80	0.82	0.82	0.82	0.81
Market-spread brake, own best	0.83	0.83	0.82	0.82	0.82
Debt brake, common (101.2%)	0.53	0.56	0.58	0.56	0.54
Debt brake, own best	0.58	0.59	0.58	0.57	0.57
Best threshold for each member					
Market-spread brake (bp)	−20	−15	−12.5	−10	−7.5
Debt brake (% of GDP)	103.5	102.7	101.2	99.7	99.7
Default under the rule (% per year)					
Credit-spread brake, 7.5 bp	0.17	0.17	0.17	0.17	0.17
Market-spread brake, common	0.49	0.40	0.31	0.24	0.17
Debt brake, common	0.01	0.01	0.03	0.17	0.30

Members identical to Italy except $b_{\text{alt}} = 4.34\,\alpha$. Gains against each member's own economy without a rule. Common: the threshold that maximizes the members' average gain. Own best: the member's best threshold among those evaluated. The best credit-spread threshold is 7.5 basis points for every member. Debt ceilings at face value.

0.04 at $\alpha = 0.8$ and 0.02 to 0.03 for the other members. Each member's own best threshold falls as its convenience yield rises, from −7.5 basis points at $\alpha = 1.4$ to −20 at $\alpha = 0.6$: a member with a higher convenience yield needs a lower threshold to hold down the credit spread at its ceiling. Even this member-specific threshold costs 0.04 percent of consumption at $\alpha = 0.6$ and 0.01 at $\alpha = 1.4$: a constant threshold cannot track a convenience yield that moves with demand over time (Section 5.3).

5.6 Robustness

The credit-spread brake is the best rule at each value of society's discount factor β_S that I consider: 0.985, 0.99 and 0.995. A more patient society gains more from holding debt down, which favors the debt brake: the credit-spread brake's lead over the best debt brake falls from 0.51 percent of consumption at $\beta_S = 0.985$ to 0.27 at 0.99 and 0.01 at 0.995. The ranking of the two spread brakes does not depend on β_S: the credit-spread brake's lead over the market-spread brake is between 0.02 and 0.03 percent of consumption at all three values. At $\beta_S = 0.995$ the three rules are close: the credit-spread brake is worth 0.65 percent of consumption, the best debt brake 0.64 and the market-spread brake 0.62.

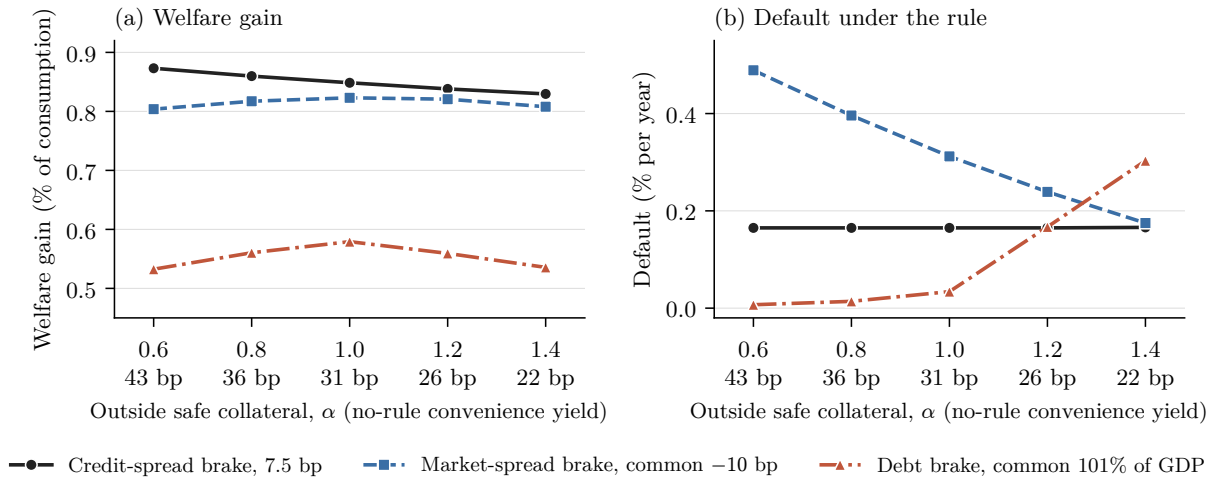


Figure 6: Welfare gain (panel a) and default (panel b) under each rule, for members that differ in outside safe collateral, $b_{\text{alt}} = 4.34\,\alpha$. Below each value of α is the member's convenience yield without a rule. Gains are against the member's own economy without a rule. Credit-spread brake at 7.5 basis points, the best threshold for every member; market-spread brake and debt brake at the common threshold that maximizes the members' average gain.

6 Conclusion

Sovereign debt carries default risk, but investors also value it for its liquidity and as collateral, and accept a lower yield to hold it. This paper asks what this convenience implies for the design of fiscal rules, and in particular which spread a spread brake should target.

Spread brakes raise welfare more than a debt brake because their ceilings follow default risk, and this gain outweighs the cost of their procyclicality. The convenience of debt matters instead for which spread the brake should read: because the market spread also moves with investors' demand for liquidity and collateral, the threshold should apply to the credit spread, all the more so in a monetary union. More generally, a fiscal rule that reads a market price responds to every force that moves that price, not only to the fiscal risk it is meant to track. The European Union's 2024 reform kept common reference values while making adjustment paths specific to each member; the results suggest that if a common anchor is kept, it should be set on default risk.

The analysis assumes that the rule is enforced: the government never borrows above the ceiling. In practice fiscal rules are often breached, and their enforcement is weak: the financial sanctions of the EU's Stability and Growth Pact have never been imposed, and markets penalize non-compliance only partially. A spread brake binds when default risk rises, which is when borrowing is most valuable to the government, so the temptation to break it may be greatest exactly when it matters most. Extending the model to rules that can be breached at a cost, as in Halac and Yared (2022) and, in a sovereign-default setting, Arce and Villa (2026), would show whether spread brakes keep their advantage when compliance is a choice.

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A Pricing risky but convenient debt

The bond prices in Sections 2 and 3 come from the problem of investors who value government bonds as collateral, as in Kaldorf and Röttger (2025, Section 4).

Investors. A unit mass of competitive investors has utility linear in consumption, discounts at the risk-free rate r_f and values the collateral services of the bonds it holds. An investor who buys b' bonds in state s pays $q(B', s)\, b'$. At the start of the next period, before income and the default decision are realized, the position provides collateral services $\zeta\, u(\theta' + b_{\text{alt}})$, where

$$\theta' = \big[1 - \kappa(B', s)\big]\, m(B', s)\, b' \tag{A.1}$$

is the pledgeable value of the position: its expected payoff net of the haircut. The haircut and the expected payoff depend on aggregate issuance B', which the investor takes as given. The bond then pays $k' = (1 - d')\{\nu + (1 - \delta)\, q(B'', s')\} + d'\, q_d(B', s')$ per unit. An investor holding b bonds with payoff k has value

$$J(b, k, s) = \max_{b'} \left\{ k\, b - q(B', s)\, b' + \frac{1}{1 + r_f} \Big[\zeta\, u(\theta' + b_{\text{alt}}) + \mathbb{E}_{s'|s} J(b', k', s') \Big] \right\}, \tag{A.2}$$

up to an endowment that keeps consumption positive. J is linear in $k\, b$, so $\partial\, \mathbb{E}_{s'|s} J(b', k', s')/\partial b' = \mathbb{E}_{s'|s} k' = m(B', s)$, and the first-order condition is

$$q(B', s) = \frac{1}{1 + r_f} \Big[m(B', s) + \big[1 - \kappa(B', s)\big]\, m(B', s)\, \zeta\, u'(\theta' + b_{\text{alt}}) \Big]. \tag{A.3}$$

In equilibrium each investor holds the debt issued, $b' = B'$, so $\theta' + b_{\text{alt}} = \Theta(B', s)$, and the condition is (26) with $\Lambda = [1 - \kappa]\, \zeta\, u'(\Theta)$. With $u(\Theta) = (1 - e^{-\zeta_2 \Theta})/\zeta_2$, $u'(\Theta) = e^{-\zeta_2 \Theta}$, which gives (24)–(25). Because services accrue before the default decision, they do not depend on whether the government defaults in that period.

Domestic holders. Domestic financial intermediaries, which households own, are a fixed share s_h of these investors. They receive that share of all payments and enjoy collateral services $s_h\, \zeta\, u(\Theta)$, the term h in (21), and both pass to households. Because the share is fixed, households' marginal utility of consumption does not enter the price.

The two-period model. With one-period bonds and no recovery, $k' = 1 - d'$ and $m = 1 - p$. The pledgeable value is $\theta = [1 - \kappa(p)](1 - p)B$, and the first-order condition gives (5) and (6).

The credit price. A claim with the same payoffs that cannot be pledged as collateral has no services term. The same first-order condition prices it at $q_P = m_P/(1 + r_f)$, where m_P is the expected payoff (23) with q_P in place of q, for both the resale value and the value of defaulted debt. The credit price reflects default and dilution risk alone. Its spread, $\text{spr}(q_P)$, is the credit spread, the model counterpart of the CDS premium.

B Data

Financial series. The financial data are daily Bloomberg series, averaged by quarter, for 2001Q1–2012Q4. For each country they comprise the five-year CDS premium and the five-year government bond yield. The risk-free benchmark is the five-year euro interest-rate swap rate. The model is calibrated to Italy. Spain, Portugal and Greece join Italy in the regressions for the scarcity slope and the common component, and Germany sets the persistence of demand. The real risk-free rate, $r_f = 0.27$ percent per quarter, is the five-year swap rate minus the five-year inflation swap rate. The inflation swap rate is available from 2004Q1, so the average is over 2004Q1–2012Q4.

Convenience yield. For country j in quarter t, the convenience yield is the CDS premium minus the market spread,

$$cy_{jt} = cds_{jt} - \left(r_{jt} - r_t^{sw}\right), \tag{B.1}$$

where r_{jt} is the bond yield and r_t^{sw} the swap rate. Its model counterpart is $\text{spr}(q_P) - \text{spr}(q)$. For Italy over 2001–2012 the CDS premium averages 79 basis points and the convenience yield 32. The CDS premium is below 20 basis points in every quarter of 2001–2007, 58 percent of the sample, and in none afterward. Over 2001–2007 the CDS premium averages 9 basis points and the convenience yield 20.

Risk-free benchmark. The benchmark is the swap rate rather than the yield on German Bunds because the Bund has a convenience yield of its own. Germany's CDS premium is close to zero, so Italy's spread over the Bund is approximately its market spread plus the Bund's convenience yield,

$$r_{IT,t} - r_{DE,t} = \left(r_{IT,t} - r_t^{sw}\right) + \left(r_t^{sw} - r_{DE,t}\right) \approx \left(r_{IT,t} - r_t^{sw}\right) + cy_{DE,t}. \tag{B.2}$$

The CDS premium minus the spread over the Bund is therefore Italy's convenience yield relative to Germany's. That relative measure falls whenever demand for German bonds rises, as in the flight to safety of 2008–10, for reasons unrelated to Italy. The model's convenience yield is measured against the risk-free rate, so its data counterpart uses the swap rate.

Debt and bonds. Debt is Eurostat consolidated gross government debt, with debt securities at market value (`gov_10q_ggfa`) and the remaining debt at nominal value, relative to quarterly GDP. Its mean over 2001–2012 is 4.59 and its standard deviation 0.24. The model counterpart is the market value of debt. The coupon $\chi = 0.0115$ is about a quarter of the issuance-weighted

average coupon on five-year Italian Treasury bonds over 2001–2012, 4.62 percent a year. Outside safe collateral $b_{\text{alt}} = 4.34$ is German government debt relative to Italian quarterly GDP.

Domestic holders. The share $s_h = 0.505$ is the 2001–2012 average share of Italian government debt securities held by private residents: resident monetary financial institutions other than the Bank of Italy, other financial institutions and other residents. The denominator is all government debt securities outstanding, at face value (Bank of Italy, *Public Finance: Borrowing Requirement and Debt*, table TCCE0200).

Fiscal series. Fiscal moments use Eurostat quarterly government finance statistics (`gov_10q_ggnfa`) and national accounts (`namq_10_gdp`) for 1999Q1–2019Q4. Public spending is general-government final consumption expenditure (P3), seasonally adjusted, relative to nominal GDP. Cyclical components are Hodrick–Prescott cycles with smoothing parameter 1600. The target correlation is that between the cycle of the spending ratio g/Y and the cycle of log real GDP. The elasticity of spending to output is $1 + b/\overline{g/Y}$, where b is the slope of a regression of the first cycle on the second.

Scarcity slope. The regression in Section 4 has the log convenience yield as its dependent variable, in quarters in which it is positive, and debt at nominal value relative to quarterly GDP and the country's own CDS premium as regressors, with country fixed effects and common time effects, for Italy, Spain, Portugal and Greece over 2001Q1–2011Q3 (158 observations). The coefficient on debt is -0.18, so the scarcity slope is 0.18. Its standard error, 0.18, is clustered by country; with only four clusters, the standard error is itself imprecise.

Common component. The variable $-cy_{jt}$, the market spread minus the CDS premium, is regressed on country fixed effects, common time effects and the CDS premium. Over 2001–2007 the common time effects account for 73 percent of its variance; with the Bund yield in place of the swap rate, they account for 83 percent. The target is the average of the two, 0.78.

Persistence of demand. The persistence target is the first-order autocorrelation of Germany's convenience yield, the swap rate minus the Bund yield plus Germany's CDS premium, over 2001–2012: 0.85. Its model counterpart is the autocorrelation of the convenience yield on a bond without default risk at debt of 4.32 times quarterly GDP, Italy's 2001–2007 mean at nominal value: 0.84.

Output loss in default. Italy did not default in the sample, so the output loss in default is set from the empirical literature, whose estimates vary widely. Borensztein and Panizza (2009) find that output growth falls in the year of a default but that the effect is short-lived. Furceri and Zdzienicka (2012) find that debt crises reduce output by about 10 percent after eight years. Trebesch and Zabel (2017) find drops in GDP of up to 10 percent in hard defaults and much smaller ones in soft defaults, and Kuvshinov and Zimmermann (2019) find that the cost is much larger

when a default is followed by a banking crisis. I target a mean loss of 10 percent of output during default and exclusion, at the upper end of these estimates.

C Proofs of Lemma 1 and Propositions 1 and 2

Lemma 1. Write $\Delta(B,y) = \psi_g\big[v(\tau y) - \phi(\tau y) - v(\tau y - B)\big] - s_h B$. For $B > 0$, strict concavity of v gives $v'(\tau y) < v'(\tau y - B)$, and $\phi' \geq 0$, so

$$\Delta_y(B,y) = \psi_g \tau\big[v'(\tau y) - v'(\tau y - B) - \phi'(\tau y)\big] < 0. \tag{C.1}$$

The gain from defaulting is therefore strictly decreasing in income. Repaying leaves public goods $g_1^R = \tau y - B$, so the government cannot repay when revenue does not cover the debt, $\tau y \leq B$; as τy falls to B, $v(\tau y - B) \to -\infty$ and $\Delta \to +\infty$. The default region is never empty, and the government defaults if and only if $y < y^*(B)$, where $y^*(B)$ solves $\Delta(B, y^*) = 0$ if a solution exists and $y^*(B) = +\infty$ otherwise, and $p(B) = F\big(y^*(B)\big)$. When y^* is interior, the implicit-function theorem applied to $\Delta(B, y^*) = 0$, with $\Delta_B = \psi_g v'(\tau y^* - B) - s_h$, gives

$$\frac{dy^*}{dB} = -\frac{\Delta_B}{\Delta_y} = \frac{\psi_g v'(\tau y^* - B) - s_h}{\big|\Delta_y(B, y^*)\big|}, \qquad p'(B) = f(y^*)\,\frac{dy^*}{dB}, \tag{C.2}$$

which is positive whenever $\psi_g v'(\tau y^* - B) > s_h$.

Proposition 1. Social welfare at debt B is $\mathcal{W}_S(B) = \mathcal{W}(B; \beta_S)$ in (3). When the ceiling binds, the government borrows $B = \bar{B}$, so $d\mathcal{W}_S/d\bar{B} = \mathcal{W}_S'(\bar{B})$. The derivative has four parts.

Step 1: current period. Debt raises proceeds $q(B)B$, which fund public goods, $g_0 = \tau y_0 + q(B)B$, while domestic holders pay their share $s_h q(B)B$. Hence

$$\frac{d}{dB}\Big[\psi_g v(g_0) - s_h q(B)B\Big] = \big[\psi_g v'(g_0) - s_h\big]\,\frac{d[q(B)B]}{dB}. \tag{C.3}$$

Step 2: marginal revenue. Differentiating the price (5),

$$\frac{d[q(B)B]}{dB} = q(B) + Bq'(B) = \frac{(1-p)(1+\Lambda) \;-\; Bp'(1+\Lambda) \;+\; (1-p)B\Lambda'}{1+r_f}. \tag{C.4}$$

The first term is the price at which the marginal bond sells. The second is the fall in the value of the existing stock because default risk rises. The third is the change in the convenience yield on that stock.

Step 3: continuation. By Lemma 1, the government repays when $y \geq y^*(B)$ and defaults otherwise, so

$$\mathbb{E}\max\{V^R, V^D\} = \int_{y^*(B)}^{\infty} V^R(B,y)\,dF(y) + \int^{y^*(B)} V^D(y)\,dF(y). \tag{C.5}$$

By Leibniz's rule,

$$\frac{d}{dB}\,\mathbb{E}\max\{V^R, V^D\} = \int_{y^*}^{\infty} \frac{\partial V^R(B,y)}{\partial B}\,dF(y) + \big[V^D(y^*) - V^R(B,y^*)\big]\,f(y^*)\frac{dy^*}{dB}. \tag{C.6}$$

The second term is zero because the government is indifferent at the cutoff. By (2), $\partial V^R/\partial B = -\big[\psi_g v'(\tau y - B) - s_h\big]$, so

$$\frac{d}{dB}\,\mathbb{E}\max\{V^R, V^D\} = -\mathbb{E}\Big[(1-d)\big\{\psi_g v'(g_1) - s_h\big\}\Big]. \tag{C.7}$$

Step 4: collateral services. Because $\Theta = \theta + b_{\text{alt}}$,

$$\frac{d}{dB}\,s_h\,\zeta\,u\big(\Theta(B)\big) = s_h\,\zeta\,u'(\Theta)\,\theta'(B). \tag{C.8}$$

Collecting terms. Combining Steps 1–4 with discount factor β_S,

$$\begin{aligned}\mathcal{W}_S'(B) = \big[\psi_g v'(g_0) - s_h\big]\frac{(1-p)(1+\Lambda) - Bp'(1+\Lambda) + (1-p)B\Lambda'}{1+r_f} \\ - \beta_S\,\mathbb{E}\Big[(1-d)\big\{\psi_g v'(g_1) - s_h\big\}\Big] + \beta_S\,s_h\,\zeta\,u'(\Theta)\,\theta'(B).\end{aligned} \tag{C.9}$$

Split $(1-p)(1+\Lambda) = (1-p) + (1-p)\Lambda$. The term $(1-p)$ and the repayment burden form S in (10): the value of the marginal bond at its credit price against the cost of repaying it. The term $-Bp'(1+\Lambda)$ forms D in (11): the loss on the existing stock from higher default risk. The terms $(1-p)\Lambda$ and $(1-p)B\Lambda'$, together with the collateral services from Step 4, form L in (12). Evaluating at $B = \bar{B}$ gives (9).

If $p' > 0$ and $\psi_g v'(g_0) > s_h$, every factor in D has a fixed sign, so $D < 0$. If $\Lambda \equiv 0$ and $u \equiv 0$, then $\Lambda' = 0$ and $u' = 0$, so $L = 0$. The government's objective is $\mathcal{W}(B;\beta_G)$, so its first-order condition is the same expression with β_G in place of β_S.

Proposition 2. The two objectives differ only in discounting,

$$\mathcal{W}_S(B) - \mathcal{W}(B;\beta_G) = (\beta_S - \beta_G)\Big[s_h\,\zeta\,u\big(\Theta(B)\big) + \mathbb{E}\max\{V^R, V^D\}\Big]. \tag{C.10}$$

Differentiating with Steps 3 and 4, and using $d\mathcal{W}(B;\beta_G)/dB = 0$ at B^G, gives (13), from which part (ii) follows directly. Part (i) of Proposition 2 follows because $\mathcal{W}_S = \mathcal{W}(\cdot;\beta_G)$ when $\beta_S = \beta_G$, so B^G maximizes social welfare. Part (iii) follows because, holding B fixed, p and θ' do not depend on b_{alt}, so $\partial[s_h\zeta u'(\Theta)\theta'(B)]/\partial b_{\text{alt}} = s_h\zeta u''(\Theta)\theta'(B)$, which is negative when $\theta' > 0$.

D The spread brake in the two-period model

Proposition 3. Treating the market spread as $p - \Lambda$, as in Section 2.7, the market-spread ceiling in state ζ solves $p(\bar{B}) - \Lambda(\bar{B},\zeta) = \bar{s}^M$, and it is unique if $p' > \partial\Lambda/\partial B$. Equivalently, it solves

$p(\bar{B}) = \bar{s}^M + \Lambda(\bar{B}, \zeta)$: a credit-spread condition with threshold (14). A credit-spread brake with fixed threshold $\bar{s}^C$ has the ceiling $\bar{B}^C$ with $p(\bar{B}^C) = \bar{s}^C$ in every state. The two ceilings coincide in state ζ if and only if $\bar{s}^C = \bar{s}^M + \Lambda(\bar{B}^C, \zeta)$. This holds in every state for a single pair $(\bar{s}^C, \bar{s}^M)$ if and only if $\Lambda(\bar{B}^C, \zeta)$ does not vary with ζ; the brakes then impose the same constraint in every state and deliver the same allocation and welfare.

Equation (15). The market-spread ceiling is defined implicitly by $G(\bar{B}, \zeta) \equiv p(\bar{B}) - \Lambda(\bar{B}, \zeta) - \bar{s}^M = 0$. By the implicit-function theorem, $d\bar{B}^M/d\zeta = -(\partial G/\partial\zeta)/(\partial G/\partial\bar{B})$, with $\partial G/\partial\zeta = -\partial\Lambda/\partial\zeta$ and $\partial G/\partial\bar{B} = p'(\bar{B}) - \partial\Lambda/\partial B > 0$.

E Italy, 2007–2012

This appendix asks whether the ranking of the rules survives the shocks Italy experienced from 2007Q1 to 2012Q4, when demand for collateral moved far more than in the calm years to which its volatility is calibrated. Income follows detrended Italian GDP. Each quarter, demand for collateral is set to the model's demand state whose convenience yield is closest to Italy's measured convenience yield. The model's convenience yield is evaluated at Italy's actual debt in that quarter. Policies and prices are those of each stationary economy; no economy is re-solved. Two features of the model limit the exercise. First, its demand states span convenience yields of about 20 to 43 basis points at Italy's debt, while Italy's measured convenience yield ranged from −50 to 105. The mapping therefore puts 2007Q4–2010Q4 and 2011Q3 in the high-demand state and 2007Q1–Q2, 2011Q2, 2011Q4 and 2012Q3–Q4 in the low-demand state. Second, the model's spreads in 2011–12 stay well below Italy's, so the model does not reproduce the sovereign-debt crisis.

Figure 7 starts each economy from its own average debt in 2007Q1: the mean of its stationary distribution of debt in Italy's 2007Q1 state of income and demand. Income was high in 2007Q1, so this exceeds the economy's unconditional mean debt. The spread brakes enter the 2009 recession with debt of about 127 percent of GDP, against 107 without a rule and 98 under the debt brake. As in Section 5.4, their ceilings fall: the credit-spread brake binds in 16 of the 24 quarters and the market-spread brake in 12. Private consumption falls to 75 percent of output, against 77 without a rule and 81 under the debt brake, and default risk peaks at 3.1 percent per quarter under the credit-spread brake and 2.2 percent under the market-spread brake. The same happens when income falls again in 2011Q4–2012Q4: the spread brakes bind, and private consumption stays near 76 percent of output, against 77 to 80 without a rule and 80 under the debt brake. The market-spread brake's one-quarter drop in 2011Q2 comes from a quarter the mapping places in low demand.

Table 8 gives the value of each rule had Italy adopted it in 2007Q1 and then faced the shocks of 2007–2012. The ranking is the same as at the benchmark: the credit-spread brake is worth 0.86 percent of consumption, the market-spread brake 0.83 and the debt brake 0.65. The columns split each gain by when it accrues. The spread brakes gain 0.40 in 2007–2012, years that carry only a fifth of the discounted weight, because starting from 105 percent of GDP they borrow up toward

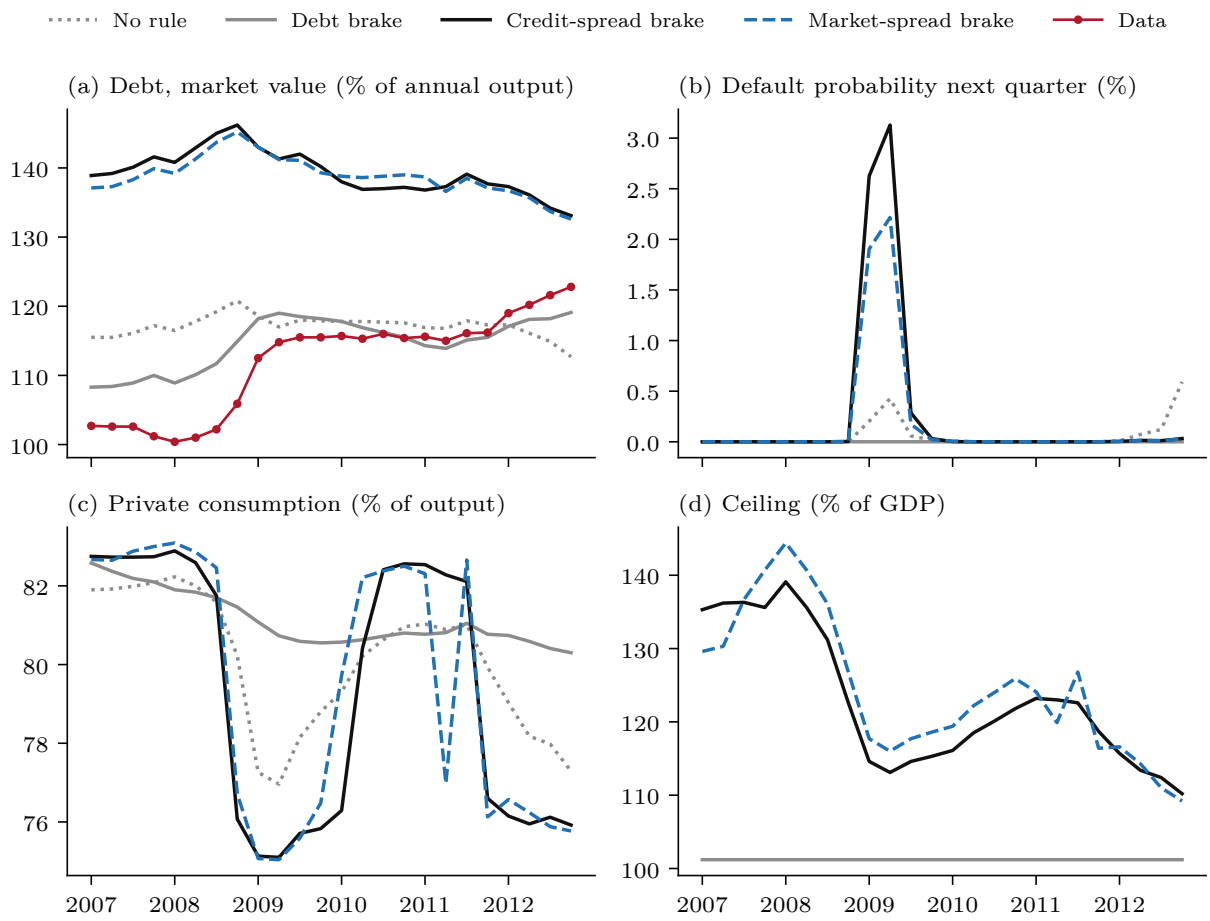


Figure 7: Each economy under Italy's shocks of 2007Q1–2012Q4. Each economy starts from its own average debt in Italy's 2007Q1 state and follows its expected choices, conditional on repayment. Panel (a) also shows Italy's debt at market value. Ceilings are face values.

their usual debt in those years. The debt brake starts above its ceiling of 101 percent of GDP and gains only 0.11 then; most of its gain comes from 2013 on, from lower default risk. The table starts every economy from the same debt. Starting each from its own average debt, as in the figure, would compare economies with different debt: the spread-brake economies carry 17 to 19 points of GDP more debt into 2007, borrowed and consumed earlier.

Table 8: Welfare gains from adopting each rule in 2007Q1 (% of consumption)

	Total	Flows, 2007–2012	From 2013 on
Debt brake	0.65	0.11	0.54
Credit-spread brake	0.86	0.40	0.46
Market-spread brake	0.83	0.40	0.43

Every economy starts from the average debt of the economy without a rule in Italy's 2007Q1 state, 105 percent of GDP, and each rule is compared with the economy without a rule. The total is split into the value of flows over 2007Q1–2012Q4 and the expected value from 2013Q1 on.

F Temporary shifts in demand for collateral

Each spread brake's stationary economy is hit by two unanticipated paths of demand for collateral, which agents learn when the episode begins. In a flight to safety, demand rises for eight quarters and then decays. Its size is set so that the convenience yield of a safe bond rises by 30 basis points on average over the eight quarters. In a withdrawal of demand, demand falls to zero for eight quarters and then recovers. Table 9 reports how each episode changes the welfare gap between the two brakes, and Figure 8 the ceilings.

In a flight to safety the market-spread ceiling loosens by eight points of GDP, but the government knows the cheap funding is temporary and does not borrow against it, so the welfare gap between the two brakes barely changes, by 0.001 percent of consumption (Table 9). In a withdrawal the market-spread ceiling falls by 38 points of GDP on impact. The convenience yield does not fall to zero: the bonds are long term, so their price still carries the convenience expected after the episode, and at lower debt the remaining convenience yield is higher. Still, the lower convenience yield raises the market spread at every level of debt, so the market spread reaches the threshold of -12.5 basis points at a debt of only 84 percent of GDP. The whole stationary distribution of debt is then above the ceiling, so the brake stops all new borrowing, while the credit-spread ceiling does not move. The gap widens by 0.014 percent of consumption.

Table 9: The two spread brakes in two episodes of demand for collateral

Episode	Change in the credit-spread brake's advantage (% of consumption)	Change in ceiling (pp of GDP)	
		Credit-spread brake	Market-spread brake
Flight to safety	+0.001	+1	+8
Withdrawal of demand	+0.014	0	−38

The first column shows how much the episode changes the credit-spread brake's welfare advantage over the market-spread brake. Welfare is measured at mean debt, median income and median demand. A positive number means the episode widens the advantage. Ceiling changes are the average over the eight quarters for the flight to safety and on impact for the withdrawal.

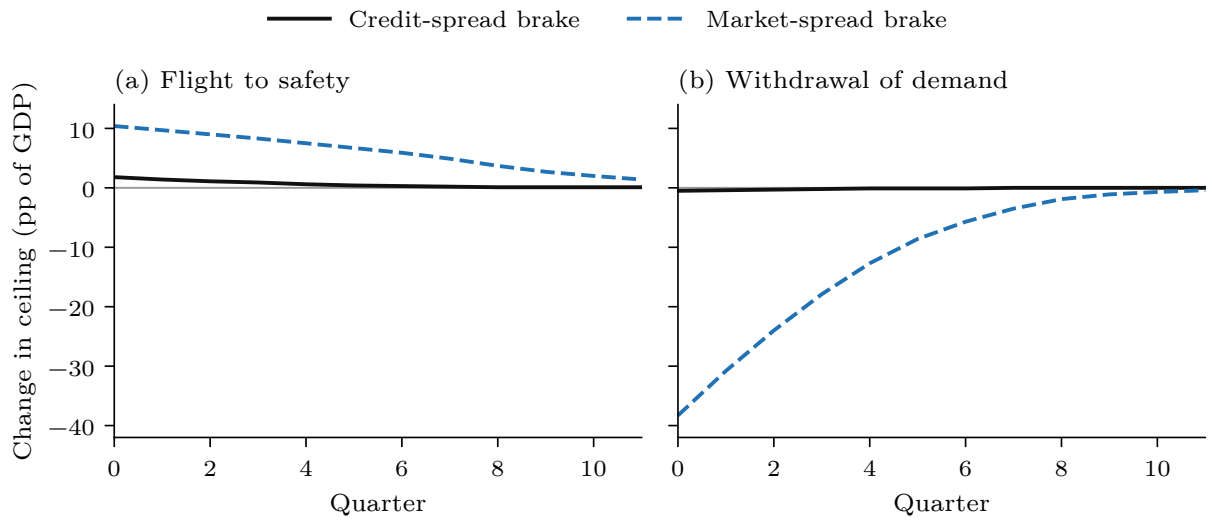


Figure 8: Change in each brake's ceiling relative to the economy without the episode, in percentage points of GDP. The episode lasts quarters 0 to 7. Averages over income and debt states, with demand states at their stationary weights.

G Welfare by income at adoption

Table 10 reports the gain from adopting each rule at the mean debt of the economy without a rule, by the state of income at adoption. Every rule is worth more when adopted in a recession, when default risk without a rule is highest. The credit-spread brake exceeds the debt brake by 0.24 to 0.29 percent of consumption, most when adopted in a recession, and exceeds the market-spread brake by 0.023 to 0.025, whatever the income at adoption.

Table 10: Welfare gain by income at adoption (% of consumption)

Income at adoption (sd from mean)	−2	−1	0	+1	+2
Debt brake	0.64	0.61	0.58	0.56	0.54
Credit-spread brake	0.93	0.89	0.85	0.82	0.79
Market-spread brake	0.90	0.87	0.82	0.79	0.76

Each rule is adopted at the mean debt of the economy without a rule (101 percent of GDP) and median demand for collateral, and compared with that economy from the same state.

H Computation

Algorithm. I compute the equilibrium that is the limit of the finite-horizon economy, following Hatchondo et al. (2022). Starting from a terminal period in which newly issued debt has no value and continuation values are zero, I iterate backward jointly on the value functions, the market and credit price schedules and the choice probabilities. In the first 400 iterations each new layer is averaged with the previous one with weight one half, which leaves the set of fixed points unchanged. The economies without a rule and with a debt brake are treated as converged when values change by less than $10^{-5}\psi_g$ and prices by less than 10^{-5}; as a check, five further iterations without averaging, and twenty further layers, stay within that tolerance. The spread-brake economies are evaluated after 4,500 iterations; at that point the remaining change in welfare is below 0.002 percent of consumption, an order of magnitude below the gap between the two spread brakes, and no reported statistic differs from the solution after 9,000 iterations.

Other equilibria. Long-term debt models can have several equilibria (Aguiar and Amador, 2020). I solve the economy without a rule again from three different starting points: an economy under a debt brake with almost no default; the credit-spread-brake economy; and default-free prices at every debt level. All three converge to the reported equilibrium except in states with debt above 175 percent of GDP and the highest income, which the economy essentially never visits; there bond prices differ by at most 2×10^{-3}. Means of debt, spreads, default and the convenience yield agree to four digits, and the welfare difference is 3×10^{-5} percent of consumption.

Grids. Income is discretized on 61 points, uniform in $\log y$ over ± 3 unconditional standard deviations. Demand for collateral follows a three-point Rouwenhorst (1995) chain with nodes at 0 and $\pm\sqrt{2}\,\sigma_\zeta$ around the mean of log demand and stationary weights $(.25, .5, .25)$. Debt takes 541 values on $[0, 8.1]$, in units of mean quarterly output.

Taste shocks. Following Gordon (2019) and Dvorkin et al. (2021), each option carries an i.i.d. Type-I extreme-value taste shock, so the government's choices are probabilities. The value is the expected maximum over the options, which adds an entropy term to their probability-weighted value. The shocks do not change budgets, feasible sets or continuation states. A numerical guard penalizes net issuance when the one-quarter default probability exceeds 0.75.

Taxes. With $\sigma = \gamma$ the tax is $T^* = \big[\bar{g} - R + \xi(y - s_h R)\big]/(1+\xi)$, $\xi = (\psi_g/\theta_c)^{1/\gamma}$, truncated to $[0, y]$.

Price cap. Prices are capped so that the implied spread is never below -105 basis points; at the calibration in Section 4 the cap does not bind.

Spread brakes. The ceiling is the debt level at which the spread crosses the threshold, and it generally lies between grid points. One option is added at the interpolated crossing; its price and continuation value are interpolated linearly between the two neighboring points.

Welfare. Policies and prices are held at their equilibrium values, and the taste-shock entropy term is omitted, so W values the government's actual choices.

I The channel decomposition

Index the economy without a rule by $j = 0$ and the economy with the rule by $j = 1$. Both start from the state in (34) and follow their own policies. Income and demand are exogenous, so each date and exogenous state (t, s) has the same probability $\pi_t(s)$ in the two economies. For any variable X, $\mathbb{E}^j_{t,s}[X]$ is its mean over economy j's distribution of debt and market access at (t, s), and $\Delta\mathbb{E}_{t,s}[X] = \mathbb{E}^1_{t,s}[X] - \mathbb{E}^0_{t,s}[X]$. Below, the subscript (t, s) is dropped.

Resources. Let $d = 1$ in default and exclusion. With market access, issuance is $I = B' - (1-\delta)B$ and domestic resources are $Z = g + c = y + (1 - s_h)(qI - \nu B)$. In default and exclusion, nothing is issued and no debt is serviced, so I and B enter (I.1)–(I.5) only with market access, and $Z = y - L(y)$. When the tax T lies strictly between 0 and y, the government splits Z optimally between g and c, so the marginal utilities of public goods and private consumption are equal, and an extra unit of resources raises flow utility by $\psi_g v'(g) = \theta_c u'_c(c)$ whichever use it goes to. This holds in 97 to 98 percent of states with market access in every economy; in the rest the tax is zero.

Channels. Split the bond price into its credit price and its convenience value, $q = q_P + (q - q_P)$. Within a (t, s), the two economies hold different debt, so there is no common debt level at which to price their issuance. S therefore values the change in issuance at one price per cell, $\bar{q}_P$, the average credit price in the economy without a rule at (t, s). This makes S a change in quantity only; any change in price goes to D. It is computed as $\bar{q}_P = \mathbb{E}^0[q_P(B', s)\, B']/\mathbb{E}^0[B']$ over the no-rule economy's states with market access at (t, s): the credit price at the debt B' chosen in each state, weighted by that debt, so states that carry more debt count more. Issuance would be the natural weight, but it is negative when the government reduces its debt. Income is the same in the two economies at each (t, s), so the change in resources is

$$\Delta\mathbb{E}[Z] = (1 - s_h)\big(\Delta\mathbb{E}[qI] - \nu\, \Delta\mathbb{E}[B]\big) - \Delta\mathbb{E}[d\, L(y)]. \tag{I.1}$$

Write $qI = q_P I + (q - q_P)I$ and add and subtract $\bar{q}_P\,\Delta\mathbb{E}[I]$:

$$\Delta\mathbb{E}[qI] = \bar{q}_P\,\Delta\mathbb{E}[I] + \big(\Delta\mathbb{E}[q_P I] - \bar{q}_P\,\Delta\mathbb{E}[I]\big) + \Delta\mathbb{E}\big[(q - q_P)\,I\big]. \tag{I.2}$$

Substituting (I.2) into (I.1) and grouping the terms gives $\Delta\mathbb{E}[Z] = S + D + L_q$ exactly, with

$$S = (1 - s_h)\big[\bar{q}_P\,\Delta\mathbb{E}[I] - \nu\,\Delta\mathbb{E}[B]\big], \tag{I.3}$$

$$D = (1 - s_h)\big[\Delta\mathbb{E}[q_P I] - \bar{q}_P\,\Delta\mathbb{E}[I]\big] - \Delta\mathbb{E}[d\,L(y)], \tag{I.4}$$

$$L_q = (1 - s_h)\,\Delta\mathbb{E}\big[(q - q_P)\,I\big]. \tag{I.5}$$

The first term of D is the higher price of debt in Table 4. The second, $-\Delta\mathbb{E}[d\,L(y)] = \mathbb{E}^0[d\,L(y)] - \mathbb{E}^1[d\,L(y)]$, is the avoided output loss, where $d = 1$ in default and exclusion: it is positive when the economy with the rule spends less time in default. The liquidity channel is $L = L_q + \Delta\mathbb{E}[\beta_S s_h \zeta u(\Theta)]$, the convenience revenue plus the change in the holders' collateral services. The channels follow Proposition 1 with one difference: D contains the avoided output loss, which is zero at the margin in the two-period model because the government is indifferent between repaying and defaulting at the default cutoff.

Units. The channels (I.3)–(I.5) are changes in resources at each (t, s), measured in goods. The welfare gain is measured in utility. Each channel is converted in three steps.

First, the change at (t, s) is multiplied by the utility value of a unit of resources there, $\theta_c u_c'(c)$. Consumption differs between the two economies, so the weight is the mean of $\theta_c u_c'(c)$ over the two economies at (t, s), denoted $\bar{M}_t(s)$. Second, the weighted changes are discounted at society's discount factor and summed over dates and states, as in the welfare function (33). This is done separately for each channel: for $X = S$, D and L_q,

$$\hat{X} = \sum_t \beta_S^t \sum_s \pi_t(s)\,\bar{M}_t(s)\,X_t(s). \tag{I.6}$$

The collateral services already enter (33) in utility, so they are summed in the same way without the weight $\bar{M}_t(s)$ and added to $\hat{L}_q$ to give L. Because every channel uses the same weight, the three sums add up to the weighted sum of $\Delta\mathbb{E}[Z]$, so the channels split the value of the total change in resources with no overlap and nothing left out. Third, each sum is expressed in the units of the welfare gain. The welfare gain is the consumption equivalent λ in (34): the fraction by which private consumption in the economy without a rule would have to rise in every date and state to make society as well off as under the rule. Raising that consumption by a fraction λ raises welfare by about $W'\lambda$, where $W' = \sum_t \beta_S^t \sum_s \pi_t(s)\,\mathbb{E}^0[\theta_c u_c'(c)\,c]$ is the welfare gain per unit of λ. A channel worth $\hat{X}$ in utility is therefore worth $\hat{X}/W'$ in units of λ, or $100\,\hat{X}/W'$ percent of consumption. With $\sigma = 2$ the exact relation is that welfare rises by $W'\lambda/(1+\lambda)$, so each channel is also multiplied by $1 + \lambda$.

Residual. The residual is the welfare gain minus $S+D+L$. The channels value only the change in mean resources at each (t, s), but each (t, s) pools histories with different debt and consumption. A rule that makes consumption less dispersed within a (t, s) raises welfare even when mean resources do not change, because utility is concave. The residual measures this gain. It is largest for the debt brake, which narrows the spread of consumption most in the quarters in which its ceiling binds.